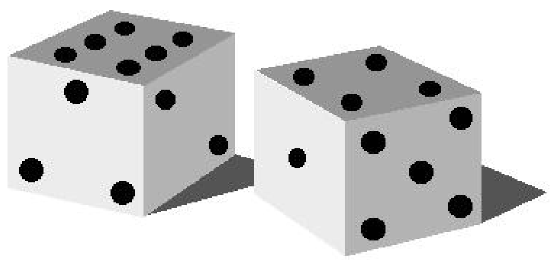
FACING THE ODDS THE MATHEMATICS OF GAMBLING



Howard J. Shaffer

Matthew N. Hall

Joni Vander Bilt

Funded in part by the Harvard Medical School Division on Addictions, the Massachusetts Council on Compulsive Gambling, the Massachusetts Department of Public Health, and the Louisiana Office for Addictive Disorders

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Contributors

Principal Writers	Matthew N. Hall and Joni Vander Bilt
Editor-in-Chief	Howard J. Shaffer
Editors	Carolyn Howard, Kathleen M. Scanlan
Education Consultant and Writer	Kathleen S. Coleman
Statistical Consultant	Richard LaBrie

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Overview of the Curriculum

Purpose

The purpose of this curriculum is to enhance students' interest in mathematics and provide the knowledge and skills that can help students to think more critically. This curriculum aims to make mathematics more meaningful to students and more relevant to their daily lives by introducing and applying concepts of number sense, data, statistics, and probability through the use of gambling- and media-related topics. With the proliferation of gambling opportunities throughout America, young people have exposure to and inherent curiosity in gambling-related matters; this curriculum uses this curiosity to present important mathematical concepts. For example, students will learn about randomness and chance as these concepts relate to probability and gambling, the probability of winning the lottery, and the use of statistics in the media and everyday life. This curriculum will also help students to develop critical thinking skills and problem-solving strategies, and then to apply these skills and strategies to media, advertising, and gambling issues. In particular, students will learn to make decisions and choices about gambling activities based on sound mathematical reasoning.

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Approach

This curriculum takes a very different approach to addictive behaviors than most traditional prevention-oriented drug and alcohol curricula. Evidence from research suggests that health classes that teach the hazards associated with drug use are not effective: students do not diminish their involvement with psychoactive substances as a result of experiencing these classes.[1] Unlike these traditional approaches, which usually emphasize the health benefits associated with avoiding addictive behaviors or attach particular values to behaviors (i.e., good vs. bad drugs), this curriculum focuses on *science* and *mathematics*, using gambling and the media as the means of presenting these concepts. For example, this program teaches probability by having students calculate their chances of winning the lottery. Unlike alcohol and substance abuse, gambling probably has not been integrated into prevention-oriented programs in most school systems in the past. Rather than focusing on the value- or health-oriented aspects of compulsive gambling and identifying gambling as another illicit activity, this curriculum reveals the mathematical realities of various gambling activities and attempts to reinforce critical, statistical, and probabilistic thinking.

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The hypothesis on which this curriculum is based is that improved critical thinking skills will decrease the unrealistic attractiveness of gambling to adolescents, delay the onset of gambling, diminish the amount of youth gambling, and minimize the harm associated with gambling. **This approach is supported by research[2] that has revealed a significant connection between gambling behavior and knowledge of mathematics: the more a student believes gambling involves skill (i.e., the lower the understanding of**

probability constructs), the more likely that student is to be a pathological gambler. This research also showed that the belief that gambling involves skill was significantly associated with higher frequency of gambling and larger wagers among adolescents. This curriculum can provide young people with the critical thinking and mathematical skills necessary to make informed decisions and choices about gambling.

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In addition, this curriculum presents some important and sometimes under-represented mathematics topics (e.g., statistics and probability). These topics are important to teach, and are expected to be more interesting to students because these topics can be related to a variety of aspects of students' everyday lives.

Benefits

It is expected that integrating the study of gambling with specific, relevant hands-on models will increase students' interest in mathematics studies. The strategy of using information related to addiction to enhance teachable moments in a traditional curriculum is quite different from conventional health education tactics that primarily help students understand the adverse effects of various addictive behaviors. **This approach can stimulate students' capacity for critical thinking and their ability to evaluate real-life situations objectively.**

Students exposed to the *Facing the Odds* curriculum will have the opportunity to practice the scientific method and critical thinking skills; more specifically, they will practice hands-on mathematics by learning about probability and gambling. Research shows that students who are more interested in mathematics have lower levels of gambling involvement, have fewer friends who gamble, and perceive gambling to be more dangerous compared to their peers who are not interested in mathematics[3]. Thus, a primary goal of ***Facing the Odds*** is to increase student interest and skill in mathematics. As a by-product of this enhanced interest and skill, one expects that student participation in potentially addictive behaviors will be delayed (for those not currently involved in these behaviors) or to diminish (for those who are currently involved in potentially addictive behaviors). Future research will be necessary to determine the efficacy of this curriculum in accomplishing these goals.

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The authors encourage teachers to think of this curriculum as a set of activities that fashions gambling as the thematic and illustrative vehicle to educate middle grades students about the essential areas of mathematics required by contemporary educational standards. Neither the study of mathematics nor this curriculum is viewed simply as a course in gambling, even though these areas overlap.

An Important Note to Teachers

You and other teachers in your district should be aware that gambling can lead to psychological, social, financial, and work- or school-related problems, and that adolescents are even more vulnerable to these problems than adults. There is a good chance that some students in your mathematics class are currently experiencing these problems and that others have family members who are experiencing these problems. It is recommended that you speak with school administrators, counselors, and nurses before teaching this curriculum so that you are prepared to offer resources to students who may be experiencing these problems.

National Council of Teachers of Mathematics Standards

This curriculum aligns with the NCTM *Principles and Standards for School Mathematics* (2000). Specifically, the activities and added practice worksheets help students meet the expectations set forth in the Data Analysis and Probability strand for grades 6–8. These expectations are stated here.

Students should…

- formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population;
- select, create, and use appropriate graphical representations of data…;
- find, use, and interpret measures of center and spread, including mean…;
- discuss and understand the correspondence between data sets and their graphical representations…;
- use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken…;
- make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit;
- use conjectures to formulate new questions and plan new studies to answer them;
- understand and use appropriate terminology to describe complementary and mutually exclusive events;
- use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations;
- compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.

A complete correlation of *Facing the Odds* to the NCTM Standards is located in the back of this binder.

Getting Started

There are 12 activities in this activity-based mathematics curriculum. Each includes teacher support and added practice worksheets (BLMs). Some activities include overhead transparencies (BLMs). Each component is discussed separately below.

> *The spirit of the* Facing the Odds *curriculum is to illustrate and enhance traditional mathematics by presenting information and analogies associated with a complex contemporary social problem that young people have identified as significant and of considerable interest.*

Activities

Each activity comprises objectives, a materials list, an estimate of the time to allow, prior understanding, an introduction (usually with a gambling connection) and discussion sections.

Teachers implementing this curriculum likely will have varying levels of expertise in the content areas presented. Therefore, the discussion material may seem too detailed for some teachers and perhaps less than sufficient for others. Teachers should take what they need from the discussions and leave the rest. Remember, the spirit of the *Facing the Odds* curriculum is to illustrate and enhance traditional mathematics by presenting information and analogies associated with a complex contemporary social problem that young people have identified as significant and of considerable interest[4]. Therefore, teachers must develop a working knowledge of each content area to fully integrate the gambling material into the more traditional mathematics curriculum. The discussion is intended to help teachers develop classroom presentations and illustrate mathematical concepts involving gambling phenomena.

Teacher Support

This component lists the relevant vocabulary and definitions. Terms that appear in the vocabulary lists are also indicated in boldfaced type in the discussion sections. The Teacher Support section also gives suggestions for ongoing assessment. These can also be used to wrap-up an activity.

> Facing the Odds *is designed to be integrated with other mathematics curricula already in place in most classrooms. Some or all of the activities can be used*.

Added Practice

Each activity features a numbered added practice worksheet that can be photocopied and assigned as homework or as a classroom assignment. Answer Keys are provided.

Overhead Transparencies

Several activities include blackline masters that can be reproduced and used as part of the lesson.

Some Important Considerations

Flexibility in Design

Facing the Odds is designed to be integrated with other mathematics curricula already in place in most classrooms. Some or all activities can be used. While they are ordered in a sequence that aligns with most common mathematics teaching practices, they can also be presented in a different order—one that fits the needs of your classroom. ***Facing the Odds* is not intended as a replacement unit for**

the Statistics and Probability strand in your mathematics course. Rather, it is a unique collection of data, statistics, and probability activities—with gambling as a central and real-world focus.

Prior Understanding

The content in this curriculum requires that students understand concepts related to fractions, decimals, and percents, including the conversion of fractions to decimals and decimals to percents. Teachers may want to review these concepts prior to presenting this curriculum.

Independent Events

When repeatedly tossing ordinary coins or throwing ordinary dice, it is generally accepted that the probability of a success on one toss or throw is independent of successes or failures on any earlier tosses or throws. However, in applying probabilities to real-world situations such as throwing free throws, getting hits in baseball, or certainly for a driver having automobile accidents in successive years, it is not clear that probabilities of successive events are fully independent. For example, some drivers are more prone to accidents than others. **Thus, when teaching about independence of successive real-world events, as in some of the examples, one assumes independence without that assumption being automatically fully justified.**

Students should be cautioned that examples used in this curriculum are for illustrative purposes only, and in actual real-world situations, the relevance of independent probabilities should be more fully explored.

State Lotteries and Gambling Venues

Legalized gambling takes many different forms and varies from state to state. While some of the examples in these activities might not mimic the types of legalized gambling in your particular state, the underlying concepts and mathematical reasoning still prevail. You might find some places where you can customize and/or extend an activity by referring to particular gaming or lottery examples in your state or community. Such customization is encouraged.

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[1] Ennet, Tobler, Ringwalt, & Flewelling, 1994; Pellow, & Jengeleski, 1991
[2] Vagge, 1996
[3] Shaffer et al., 1995a; Shaffer et al., 1995b
[4] e.g., Shaffer et al., 1995

Background of the Curriculum

This mathematics curriculum involving number sense, data, statistics, and probability was developed in an attempt to increase young people's mathematics literacy while concurrently preventing or reducing their participation in a potentially addictive behavior. Using a randomly selected sample of middle school students from the Merrimack Valley area of Massachusetts, research revealed that a range of students were interested in learning about science, mathematics, and the addictive behaviors.[5] Consequently, the authors integrated the study of gambling with mathematics to yield an innovative and socially relevant curriculum for middle school students. By integrating these two apparently distinct areas, the authors believe that the intellectual, psychological, and social health of America's young people can be improved. Alcohol, tobacco, other drugs, and gambling have the potential to undermine the developmental integrity of America's young people; however, scientific and mathematical literacy has the potential to promote the healthy development of America's youth.

Gambling in the United States

During the 1980s and 1990s, the proliferation of American gambling has been extraordinary. In addition to the recent availability of riverboat, Native American, and urban casinos, the lottery has become a staple of American gambling. In spite of warnings from scholars[6] and social policy makers about the potential adverse consequences, state-sponsored gambling's ability to generate revenue without increasing taxes has shifted American morality not only to tolerate but to endorse legalized gambling. Between 1974 and 1996, the total amount of money legally wagered in the United States increased from $17.3 billion to $586.5 billion.[7] Between 1975 and 1985, the national per capita sales of lottery products alone increased from $20 to $97.[8] By 1994, the national per capita lottery expenditure had risen to $120.[9] Figure 1 illustrates the increase of per capita lottery expenditure during the last two decades.

Problem Gambling Among Adolescents

Young people in the United States today are the first generation in more than a century to grow up in a climate where gambling is not only legal, but avidly encouraged and culturally approved; lottery numbers are often reported with headline news and the weather. Many new types of gambling have become available to young people during the past several decades. Though officially gambling is illegal to those under 18 years of age in most states, many forms of gambling are easily accessible to those who wish to gamble. Adolescents have responded to this new climate of gambling, not surprisingly, by gambling more today than ever before. In fact, studies have shown that the prevalence of pathological gambling is higher among adolescents than in the general adult population.[10]

Young people in the United States today are the first generation in more than a century to grow up in a climate where gambling is not only legal, but avidly encouraged and culturally approved; lottery numbers are often reported with headline news and the weather.

There are many ways to gamble. Two students playing basketball in the gym after school could bet a dollar on whether one would make a three-point shot. A tenth-grade student could be forming a betting pool for wagers on the outcome of sporting events. Students could be playing poker on Saturday nights for pennies, or for bigger money. An eighth-grade student could try to buy a lottery ticket with leftover change from buying a pint of ice cream at the local convenience store. A group of high school seniors could drive to a casino and take a chance on being admitted. Although the legal age to enter a casino is 21, 64% of underage students at one Atlantic City high school reported gambling at casinos.[11] Some students will gamble occasionally, some regularly, but approximately 3.9% are gambling at a level to be considered pathological gamblers.[12]

Two studies examined the levels of involvement of Massachusetts' adolescents in various illicit activities, including the lottery.[13] As Figure 2 illustrates, of six illicit activities (alcohol use, cigarette smoking, cocaine use, lottery gambling, marijuana use, narcotics use) investigated among students in grades 7 through 12, the lifetime prevalence of involvement with the lottery is exceeded only by the lifetime prevalence of alcohol use.[14] A similar pattern emerges for current involvement (i.e., within the past 30 days) with these six activities (see Figure 3). Current prevalence rates are more accurate indicators of existing psychosocial problems than are lifetime rates.

Another study of middle-grades students investigated gambling activity in two categories: gambling on the lottery and gambling on other events (e.g., sports betting, card games). As Figures 4 and 5 illustrate, this study revealed that among younger children, both current and lifetime rates of involvement with gambling on events other than the lottery are higher than the rates of involvement with seven other illicit activities (those previously mentioned, plus use of inhalants and use of stimulants), including the lottery.[15] These adolescents' involvement in sports betting, card games, and other gambling activities exceeds their involvement in the lottery and may, in fact, provide the gateway activity not only to other gambling experiences but to substance abuse as well.

Pathological Adolescent Gamblers

Researchers also examined the adverse social and emotional consequences of gambling for adolescents.[16] The pathological student gamblers in this study's sample experienced a variety of problems, which is detailed in the following chart.

Of students considered to be pathological gamblers …

89% were preoccupied by gambling;
85% chased their losses;
79% lied to conceal their gambling;
79% committed illegal acts to finance gambling;
71% had problems at home, work, or school due to gambling;
71% got in trouble at work or school because of gambling;
68% neglected their home, work, or school obligations for at least two consecutive days because of gambling;
68% jeopardized or lost a significant relationship, job, or educational or career opportunity because of gambling;
64% felt pressure to gamble when they did not gamble;
64% were unable to stop when they wanted;
61% felt social, psychological, financial pressure to increase the amount gambled;
54% had been arrested because of gambling;
43% felt guilty about their gambling; and
39% had sought help for their gambling problems.

Shaffer, Hall, Walsh, & Vander Bilt (1995)

These adolescents also reported symptoms that represent neuro-adaptive patterns, much like those observed among people with substance dependence disorders. For example, 69% reported that the same amount of gambling had less effect on them than it had previously, and 83% increased the amount they were gambling to get the same effect as they had experienced at a lower level of betting.

Both of these symptoms represent tolerance symptoms. Like the chemically dependent who have withdrawal signs and symptoms, pathological gamblers can make their symptoms go away by gambling again. Of the pathological gamblers in this sample, 77% got restless, irritable, and had difficulty concentrating when they stopped gambling, and 54% continued to gamble to make withdrawal symptoms go away. Like psychoactive stimulant abuse, gambling influences the central nervous system in a powerful way. Adolescent pathological gamblers can escape feelings of depression by continuing to gamble: 79% of the pathological gamblers in this sample reported that they gamble as a way of escaping problems or relieving feelings of helplessness, guilt, anxiety, or depression.

However, pathological gamblers are not the only people who experience problems related to their gambling. For example, 28% of youthful gamblers who do not meet diagnostic criteria report "chasing" their losses. An alarming 16% of these young people report having experienced some physiological symptoms (i.e., tolerance) related to their gambling. For non-pathological student gamblers (i.e., students unlikely to be identified by screening or other diagnostic tools) the prevalence of psychological distress related to gambling is similar to the rates of alcohol dependence.

Next Steps

Studies show that most pathological gamblers start gambling when they are adolescents[17]. Considering that both the opportunities for gambling and the prevalence of gambling are on the rise, it is necessary to provide young students with the tools they need to make wise decisions about their gambling activities. Although only a small percentage of adolescents will struggle with pathological gambling, a much larger percentage of adolescents will face some level of psychological distress due to gambling. One of the best strategies for prevention and early intervention for the majority of adolescents could be the development of a sharp inquiring mind and the application of a few mathematical principles. One of the seven recommendations that emerged from the North American Think Tank on Youth Gambling Issues was that curricula and programs be developed to educate parents, children, and teachers about the issue of youth gambling.[18] *Facing the Odds* represents a first step toward that goal.

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[5] Shaffer et al., 1995a
[6] Eadington, 1992; Shaffer, 1989
[7] Commission on the Review of the National Policy Towards Gambling, 1976; Christiansen, 1997
[8] Clotfelter & Cook, 1989
[9] McQueen, 1997
[10] Shaffer, Hall, & Vander Bilt, 1999
[11] Arcuri, Lester & Smith, 1985
[12] Shaffer, Hall, & Vander Bilt, 1999
[13] Shaffer, 1994; Shaffer et al., 1995a
[14] Shaffer, 1994
[15] Shaffer et al., 1995a
[16] Shaffer, Hall, Walsh, & Vander Bilt, 1995
[17] Gaboury & Ladouceur, 1993
[18] Shaffer, George & Cummings, 1995, p. 2

Students’ Introduction to the Curriculum

Objectives

- Identify areas of student interest in applying mathematics to gambling and everyday life
- Provide an opportunity for students to express their feelings and opinion about having input on curriculum content

Materials	notebook or paper for use as a mathematics journal, pencils
Time	30–45 minutes
Math Idea	Students who are given the opportunity to reflect on topics and the opportunity to satisfy their own curiosity regarding mathematical applications will have the germinal experience and excitement of mathematical understanding. Students with a desire for knowledge, who feel that learning and inquiry satisfy personal interests, will be bolstered and motivated to actively question and seek out answers to their own questions.

Introduction: Gambling Connection

Begin with a 2- or 3-minute introduction about the prevalence of gambling behavior and problem gambling among youth in schools, by family members in the home, and social concerns relevant to the students' locale is suggested. This information can be obtained from the introduction to this curriculum.

You may want to cite current media stories related to gambling-related issues that students can appreciate. It is important that the topics be real and pertinent to each student—asking students to think about (but not identify) someone they know who gambles allows seemingly academic topics to become alive, real and pertinent. Also, several graphs are included as BLMs that can be used as overhead transparencies and serve as the basis of discussion.

Exercise

Brainstorm a list of mathematics topics that are relevant to everyday life. Ask, *What questions come to mind that you think mathematics could help you solve? What questions come to mind when you think about gambling?*

Give students two or three minutes to brainstorm individually and develop ideas on their own. Then ask volunteers to describe an area of interest. On the chalkboard or overhead transparency, record each topic. Repeat the topics as you are writing. These two activities are expected to validate significantly the process of brainstorming.

Once you have recorded twenty or more topics, congratulate the class for their effort. With this exercise, you will have a solid pool of information from which to develop a core-interests list. Inform the class that while all the information on the board is valuable, there is too much to possibly cover all of it. In order to focus on topics of most interest, students will vote for the top five topics. (The top five choices are expected to be covered in the existing mathematics curriculum or in this curriculum.)

- Ask students to select the five topics that are of most interest to them for future study.
- Complete a tally chart showing each topic and its number of votes.
- Identify the five topics with the most votes.

Discussion

Verify that each student has at least one interest listed in the top five. If there are students who are not interested in any of the top five, you may ask them for their top two choices and let them know that these will be covered over the course of the year. In the unlikely case that none of a student's choices is covered in the curriculum, you have the opportunity to demonstrate how important it is that everyone has a special interest covered in mathematics.

After acknowledging that a special effort will have to be made to meet this student's needs, the teacher and student(s) will create a project that meets this particular student's interest needs. Discussing this with the class is expected to bolster student interest, commitment and involvement as well as providing a first-hand experience that the teacher is motivated and willing to meet students' needs and interests.

Finally, a typed copy of the selected topics can be prepared and given students to be put in their math journal. Also, a large print copy placed in plain view for the duration of the curriculum will allow students to see their progress as their identified areas of interest are addressed. It is a potent teaching strategy to point out at the end of the curriculum the posted list and show how over the course of the curriculum the individual science interests of every student in the class were addressed. Citing each student in an informal manner and the advances they made, especially those who became increasingly interested in science over the course of the year, is expected to cement this key step in the scientific method: identifying areas of interest.

Teacher Support

Background

It is assumed that teachers delivering this curriculum will have made themselves amply comfortable with the material included in the Introduction and the Curriculum Overview.

As educators, we are constantly balancing teaching with our own learning, attempting to integrate new information with existing knowledge, all the while trying to stay aware of our own biases, beliefs, prejudices and judgements. This exercise may challenge some teachers' beliefs regarding the extent to which student involvement in curriculum development is appropriate, or to what extent students should be self-directing regarding what it is they want to learn.

The scientific method, in its truest sense, starts with the curiosity of the investigator. For students to become engaged they must feel and believe that it is their curiosity that is being supported by teachers, parents, siblings and peers. In this context students must believe that they are being given the opportunity to develop tools that will allow them to continue to explore their worlds through curious eyes long after middle school.

Ongoing Assessment

In many cases, this exercise may be the first opportunity for students to participate in curriculum development. As is the case with any "first experience" unfamiliar feelings may arise with the students. Take time to inquire about these feelings and the experience the students have had. Ask questions such as these:

- *How does it feel being given the opportunity to share your interests?*

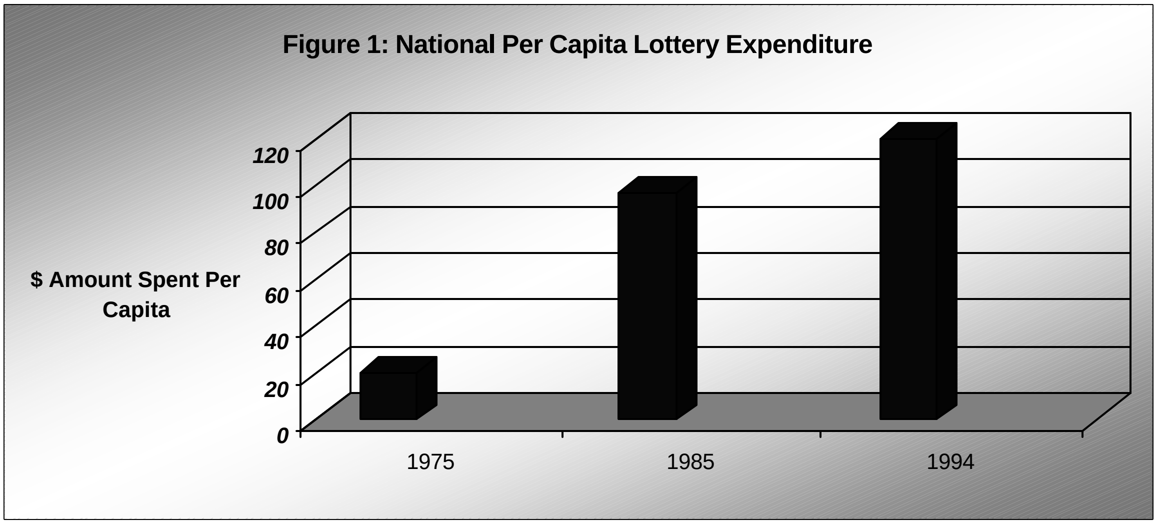
- *Are there any topics that we are going to study that you are interested in now that you were not aware of before we did this exercise?*

- *Would you like to do this exercise in other courses or do you prefer the teacher to just tell you what we are going to be learning?*

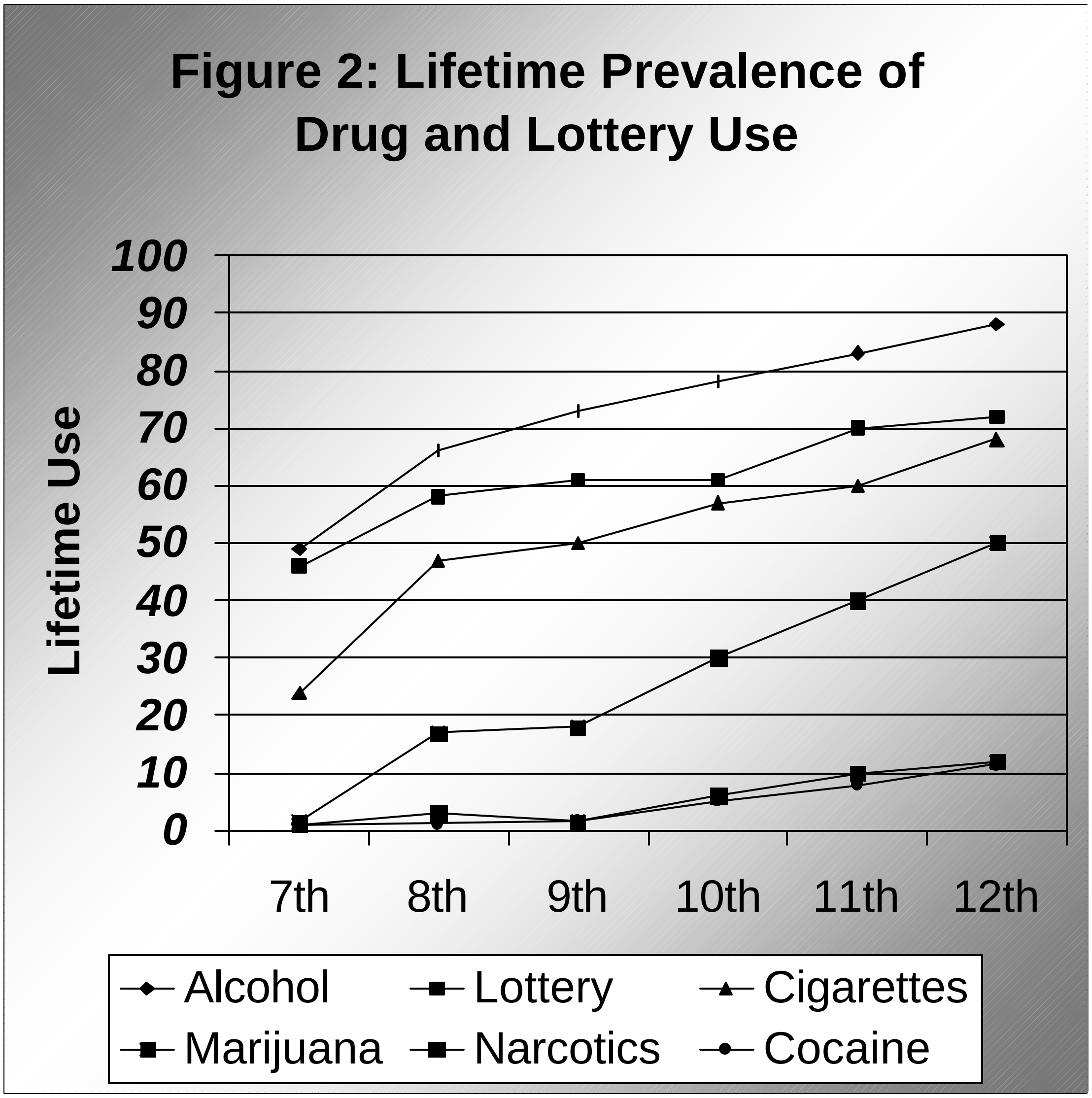
- *Do you feel more curious about what we are going to do in mathematics this year because of doing this exercise?*

- *What one thing did you like most about doing this exercise?*

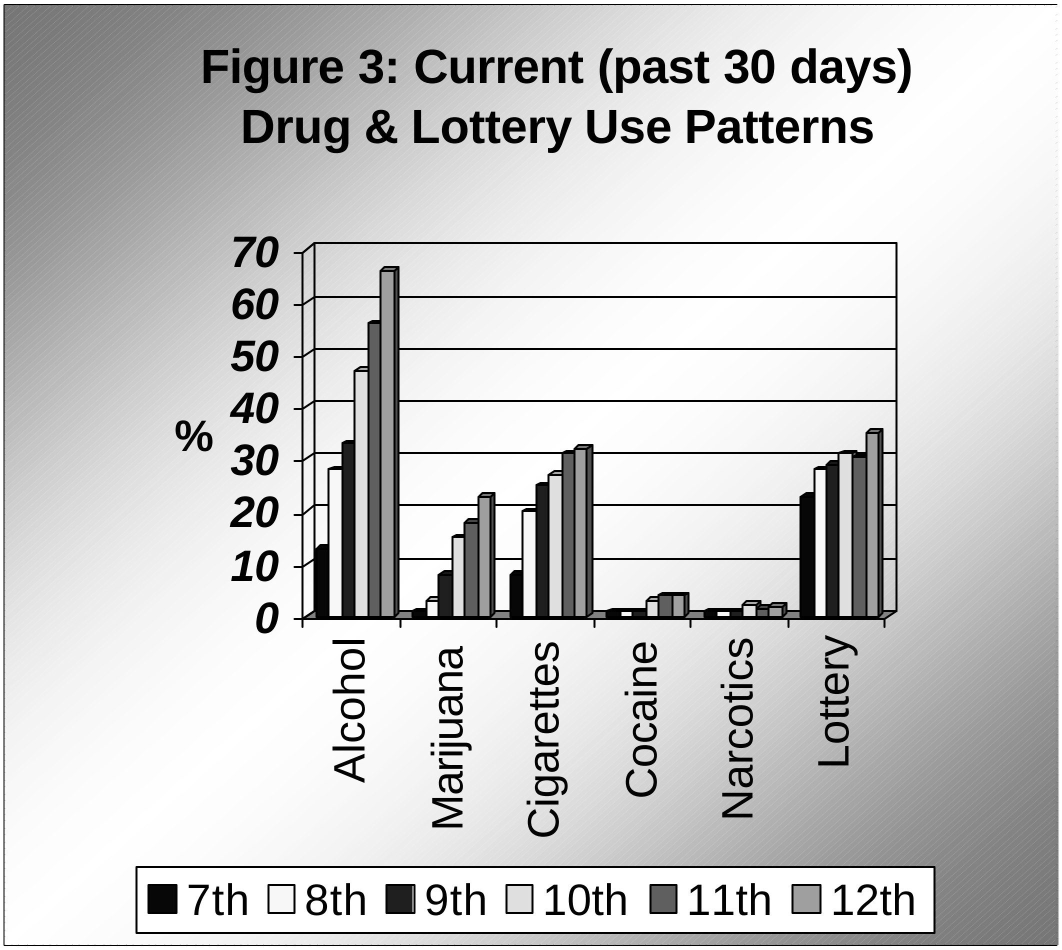
Blackline Master Students' Introduction



Blackline Master Students' Introduction



Blackline Master Students' Introduction



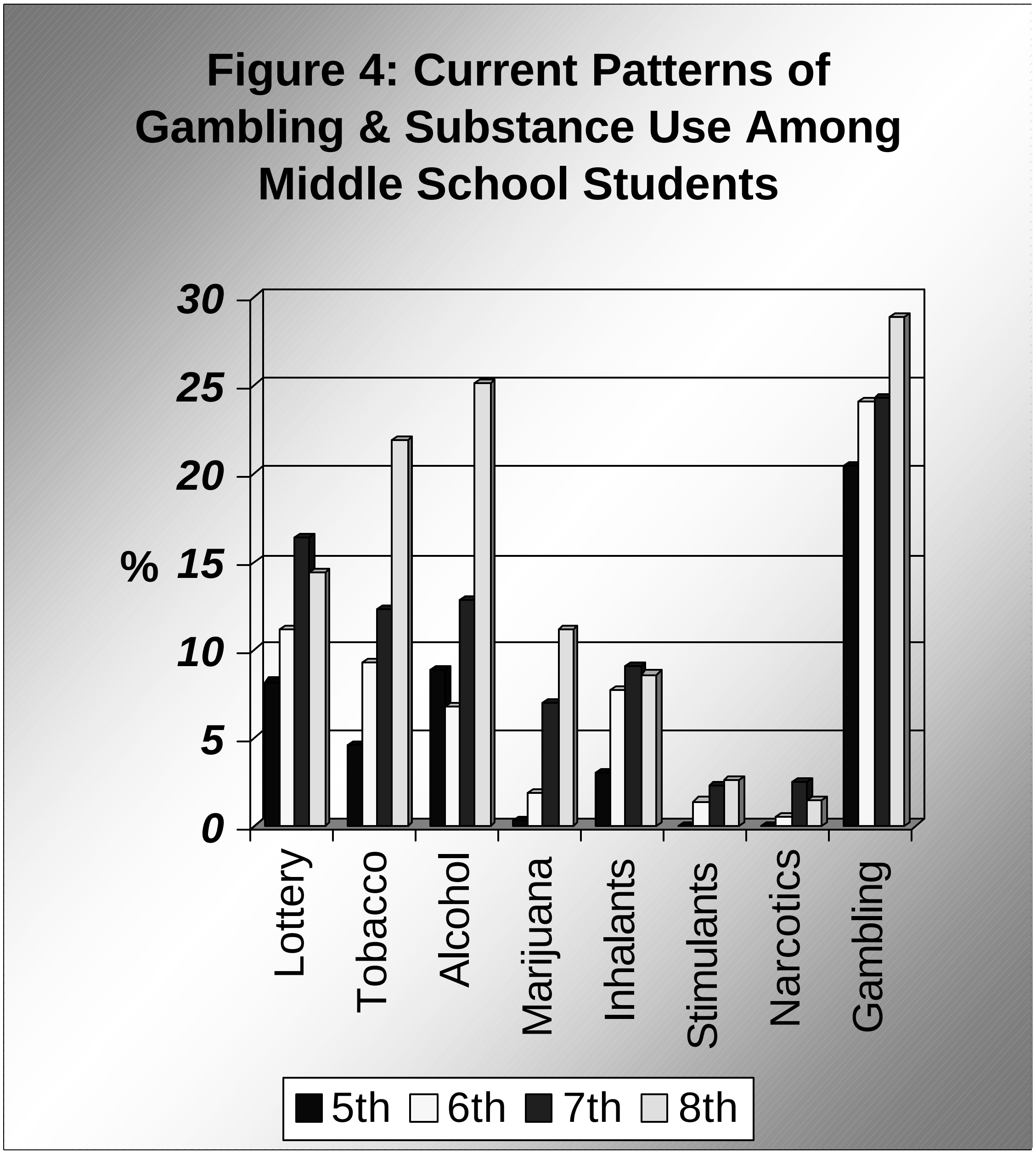
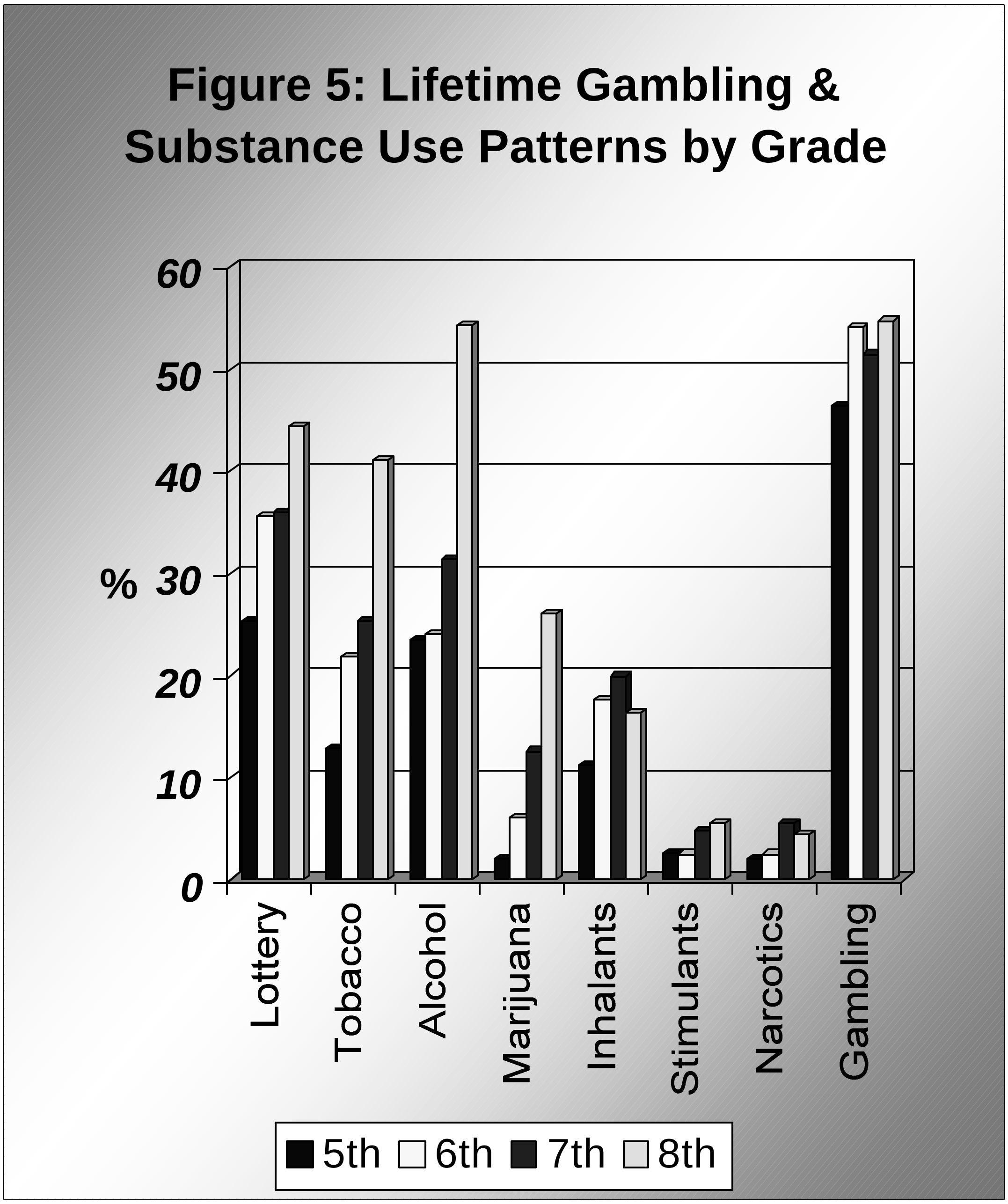
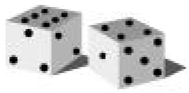


Figure 4: Current Patterns of Gambling & Substance Use Among Middle School Students

Blackline Master Students' Introduction





Activity 1 Thinking About Averages

Objectives

- Apply the concepts of mean, median, and mode
- Understand the concept of sampling
- Collect, organize, and display data in a variety of ways
- Use observations about differences between samples to make conjectures about the populations involved

Materials	paper, pencils, calculators
Time	30–45 minutes
Math Idea	The word *average* often connotes “typical,” “common,” “middle,” or “midpoint.” However, statistical averages have a more specific meaning.

Prior Understanding

Students should know how to find the mean, median, and mode for a set of values. They should also know how to calculate with decimals and convert from one measurement unit to another.

Introduction

The following scenario may be used as an introduction to the problem of thinking about averages.

A wealthy couple is planning a dinner party for 5 guests. Their ages are 89, 92, 17, 2, and 2. The couple gives a list of the guests and their ages to the butler, the disc jockey (DJ) in charge of entertainment, and the cook so that proper arrangements can be made. Based on "averages," what would you predict would be served at dinner? What kind of music should be played? Would you serve after-dinner drinks? Why or why not?

Discussion

After discussion of the scenario, tell students that at the party everyone was served strained peas at dinner accompanied by the latest CD from The Backstreet Boys, followed by cognac. Discuss how the three members of the staff could each have arrived at a different idea based on "averages."

Students might realize that the butler looked at the guest list and calculated the mean age: (89 + 92 + 17 + 2 + 2)/5 = 202/5 = 40.4, so served cognac after dinner. The DJ used the **median** age, 17, as the average, so picked appropriate music. The cook used the **mode** or most common age, 2, as the average and served strained peas. Although this scenario would probably not occur in real life, students should be aware of the different ways to discuss averages.

Exercise 1

> Help students develop a simple data chart to collect information that could be used to find the “average” height for your class. Divide students into groups. Let each group decide how they will choose a sample and collect and record the data. Then have each group calculate the mean, median, and mode for its sample.

Discussion

Briefly, discuss ideas for a data collection chart. Two columns, one with a name, the other with a height, are sufficient. Students should be aware of the advantage of organizing their **data** in a manner that allows for easy calculation.

Each group of students will most likely choose a different **sample** (will ask a different combination of students), so expect that students will come up with a diverse range of **means**, **medians**, and **modes**. Allow 10 minutes for students to collect their data; some students will collect more **data** than others.

Discuss the problem of making calculations with heights that are recorded in mixed units, such as 5'2" and 4'8". If the heights were first converted into one unit, such as inches, the task is easier. After calculating the **mean**, **median**, and **mode**, students can convert each average back into feet and inches.

Help students realize that a **sample** is a small portion taken from the entire population, which would be every individual in the group of all possible observations. Often it is unrealistic to examine every member of a population, especially if that population is very large. In addition to being time-consuming, evaluating an entire population is often too expensive. For practical purposes, studies are usually conducted by selecting a **sample** from a larger population. To ensure that this smaller sample is representative of the larger population, the **sample** should be selected according to a random process. In random processes, each member of the population has the same chance of being selected.

Discuss whether students selected their **sample**s randomly. For example, a random sample might be every fifth name off the class roster listed alphabetically. Most students probably selected their friends, those in their group, or those who were sitting nearest to them. This might be a *biased sample*; if an 8th grader who is 5'6" sampled his or her friends, he or she may be more likely to have friends who also are tall.

Exercise 2

Compile data for the entire class on the board and have students find the mean, median, and modal heights for the class. Discuss differences between the total class results and the results from their samples. Let students speculate as to how the sample they used might have influenced their results.

Discussion

Students should realize that the only way to calculate the precise **mean**, **median**, and **mode** for the class is to include every student in the **sample**. However, if a random sample was chosen, the three measures they obtained might be good *estimates* for the class.

Activity 1 Thinking About Averages Teacher Support

Vocabulary

data information, often in the form of facts or figures obtained from experiments or surveys, used as a basis for making calculations or drawing conclusions

distribution the spread of statistics within known or possible limits

mean the arithmetic average of a set of numbers

median the middle value in a set of values that have been arranged in ascending or descending order; the midpoint, or the value in a distribution above and below which 50% of the values lie

mode the number that occurs most frequently in a set of numbers

sampling the process of selecting a group of people to be used as representative of an entire population

Ongoing Assessment

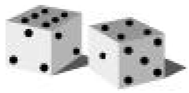
For any one mean, there are many possible **distributions** that could produce that mean. Have students estimate the **mean** for each of these three **distributions**:

(a) all the values are between 95 and 105

(b) half the values are around 50 and the other half are around 150

(c) one-quarter of the values are 0, half are near 50, and one-quarter are around 300

(All three distributions have the same mean—100; however, the other characteristics of these distributions are very different.)



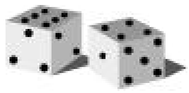
Added Practice 1 Thinking About Averages

Name ______________________________Date______________________

1. Suppose you want to survey residents of Baton Rouge to find out their favorite radio station. Would it make sense to sample every person who lives in Baton Rouge for your survey? Why or why not?

2. If you ask a sample of people living in Baton Rouge what their favorite radio station is, what could you do to select a sample whose answers would reflect that of the larger population?

3. Two American women, Florence Griffith Joyner and "FleetFoot," are competing in the 100-meter dash. Florence Griffith Joyner's past times were 11.08, 10.81, 10.75, 10.62 and 10.49 seconds, for an average of 10.75 seconds. FleetFoot's past times were 11.51, 11.25, and 11.89 seconds, for an average of 11.55 seconds. To calculate the qualifying time for the team, the officials find the mean time of these two runners. What is the team's mean time? What is the team's median time? What is the team's mode time?



Answer Key

Added Practice 1 Thinking About Averages

1. Students might say that it would be too time-consuming and/or too expensive to survey the entire population of Baton Rouge. By the time they finished asking everyone, the radio stations could have changed formats or no longer exist.

2. Students should address ideas about who to include or not include in the sample and how to pick a random sample rather than a convenient one (such as friends and relatives). Students should also examine whether the members of the selected group have any common characteristics, such as age, education level, income.

3. The average time for the team is 11.05 seconds. Students should add the values representing both runners' previous times and divide the sum by 8:

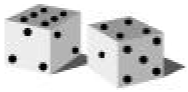
11.08 + 10.81 + 10.75 + 10.62 + 10.49 + 11.51 + 11.25 + 11.89 = 88.4; 88.4/8 = 11.05

A common mistake with this type of question is to average the averages, or

10.75 + 11.55 = 22.3; 22.3/2 = 11.15 seconds.

If both women had run the same number of races (four), then you could find the average time by averaging the two averages. However, Joyner ran five races and "FleetFoot" ran three races. To find Joyner's average (10.75), the five times were “condensed” into one; to find Fleetfoot's average (11.55) three times were "condensed" into one. Each of Joyner's five times is represented less in her average than each of Fleetfoot's three times is represented in her average. In other words, each of Joyner's times represents one fifth of her average, but each of Fleetfoot's times represents one third (a greater portion) of her average. Thus, if you average the averages, Fleetfoot's times are "weighted" more, or represent more, than Joyner's do in the final average.

To find the team’s median time, first arrange the times in order: 10.49, 10.62, 10.75, 10.81, 11.08, 11.25, 11.51, 11.89. The middle value lies between 10.81 and 11.08: (10.81 + 11.08)/2 = 21.89/2= 10.95. There is no mode.



Activity 2 Number Sense

Objectives

- Develop an understanding of large numbers
- Use mathematics to evaluate media messages and gambling opportunities

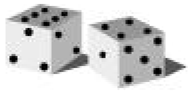
Materials	paper, pencils, calculators
Time	45 minutes
Math Idea	Understanding mathematical and statistical concepts is vital to a wide range of topics, including personal finances, sports, insurance, risk/reward trade-off of everyday activities, diet and medical claims, and elections.

Prior Understanding

Students should know how to convert time units and use a calculator to add, subtract, multiply, and divide.

Introduction

General number sense is very helpful in evaluating gambling opportunities and media claims. For some students it is hard to distinguish between millions, billions, and trillions—making these numbers meaningless. Give students the following facts as an introduction to the activity.



· *In 1999, the federal government spent $1.72 trillion on public finance.*

· *In 1995, state lotteries produced more than $11 billion in revenues.*

· *According to a 1994 gambling study, as many as 1.3 million teenagers had some form of problem-gambling behavior.*

Discussion

Ask students to describe how long they think 1.72 **trillion** seconds is compared to 11 **billion** seconds or 1.3 million seconds. Have them calculate the number of centuries in 1.72 **trillion** seconds; the number of years in 11 **billion** seconds; and the number of days in 1.3 million seconds.

Putting the numbers into a context that students understand will help them get a better conceptual grasp of these large numbers:

- 1.3 million sec = 21666.67 min = 361.1 hr = 15.05 days
- 11 **billion** sec = 183333333.33 min = 3055555.56 hr = 127314.8 days = 348.8 years
- 1.72 **trillion** sec = 28666666666.667 min = 477777777.78 hr = 19907407.41 days = 54540.8 years = 545.4 centuries

Exercise 1

Distribute copies of BLM 2A “Number Sense” to students. Have them try to match the categories in column A with the appropriate values in column B.

Discussion

It is helpful for students to develop an awareness of some of the characteristics of their society and the world in a quantifiable way. By completing BLM 2A “Number Sense” students can evaluate their knowledge of large numbers and how these numbers related to different areas of society.

Exercise 2

Divide the class into groups. Distribute a copy of BLM 2B “Time Flies” to each group. Have students use the given information to estimate how much time a teenager who smokes would lose from his or her lifetime.

Discussion

The purpose of completing BLM 2B “Time Flies” is to give students the opportunity to see how different rates relate to each other and how a rate on one scale (daily rate) relates to the same rate on a larger scale (over a lifetime rate). When all the groups are finished, check to see if all have arrived at the same conclusion.

Activity 2 Number Sense Teacher Support

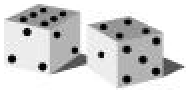
Vocabulary

billion one thousand million; 1,000,000,000

trillion one thousand billion; one million million; 1,000,000,000,0000

Ongoing Assessment

Have students find examples of large numbers used in newspaper or magazine articles. Have them convert the numbers to some meaningful frame of reference, such as days, weeks, years. Invite them to make bar graphs to illustrate their calculations. *(Examples and graphs will vary.)*



Added Practice 2 Number Sense

Name ______________________________Date______________________

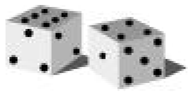
The following data represents population figures and lottery expenditures for the six New England States in 1997.

New England State	**Population**	**Lottery Expenditure** (dollars)	**Total Gambling Expenditure** (dollars)	**Lottery Expenditure** (per capita)	**Total Gambling Expenditure** (per capita)
Maine	1,242,000	185,000,000	260,000,000		
New Hampshire	1,173,000	176,655,620	550,000,000		
Vermont	589,000	77,323,314	78,500,000		
Massachusetts	6,118,000	3,100,000,000	3,800,000,000		
Rhode Island	987,000	548,715,864	600,000,000		
Connecticut	3,270,000	500,000,000	7,500,000,000		
Totals					

Round answers to the nearest hundredth where needed.

1. Fill in the Lottery Expenditure (per capita) column by calculating the mean amount spent per person on the Lottery for each of the six New England states listed in the table.

2. Calculate the total Population, the total amount spent on the Lottery, and the total amount spent on all types of gambling for these New England states. Write your answers in the table.



3. Calculate the per capita expenditure (mean amount spent per person) on the Lottery in New England for 1997. Write your answer in the table.

4. Rank the states from highest to lowest per capita Lottery expenditure. Why do you think the differences among the states are so big?

1. 4.

2. 5.

3. 6.

5. Fill in the Total Gambling Expenditure (per capita) column by calculating the mean amount spent per person on gambling for each New England state.

6. Calculate the per capita expenditure on gambling for New England. Write your answer in the table. How does this number compare to the per capita expenditure on the Lottery for New England?

7. Rank the states from highest to lowest total expenditures on gambling. How do these rankings compare to the per capita lottery rankings? What do you think accounts for the differences? Why might figures for state per capita expenditures on gambling may be misleading?

1. 4.

2. 5.

3. 6.

8. How much does the average New England resident spend on the lottery *per day*?

9. How much does the average New England resident spend on gambling *per day*?

Answer Key Added Practice 2 Number Sense

1. Students should divide each state's Lottery expenditure by the state's population.

2. Students should add to find the totals for the first three columns.

3. Students should divide the total Lottery expenditure for New England by the total population for New England.

New England State	**Population**	**Lottery Expenditure** (dollars)	**Total Gambling Expenditure** (dollars)	**Lottery Expenditure** (per capita)	**Total Gambling Expenditure** (per capita)
Maine	1,242,000	185,000,000	260,000,000	$148.95	$209.34
New Hampshire	1,173,000	176,655,620	550,000,000	$150.60	$468.89
Vermont	589,000	77,323,314	78,500,000	$131.28	$115.45
Massachusetts	6,118,000	3,100,000,000	3,800,000,000	$506.70	$621.12
Rhode Island	987,000	548,715,864	600,000,000	$555.94	$607.91
Connecticut	3,270,000	500,000,000	7,500,000,000	$152.91	$2293.58
Totals	13,379,000	4,587,694,798	12,778,000,000	$342.90	$955.08

4. The states in order from highest to lowest per capita expenditure on the Lottery are:

1. Rhode Island
2. Massachusetts
3. Connecticut
4. New Hampshire
5. Maine
6. Vermont

The differences among the states could be attributable to the advertising budgets and techniques of the lotteries, the types of lottery games the states offer, how often new games are introduced, the income levels of state residents, the general attitudes toward gambling in the different states, or other factors.

5. Students should divide each state's total gambling expenditure by the state's population.

6. Students should divide the total gambling expenditure for New England by the total New England population.

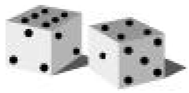
7. The states in order from highest to lowest per capita expenditure on gambling are:

1. Connecticut	4. New Hampshire
2. Massachusetts	5. Maine
3. Rhode Island	6. Vermont

Only Vermont has the same ranking (6th) in both per capita Lottery and per capita gambling expenditures. The most extreme difference between Lottery and overall gambling expenditures occurs in Connecticut: whereas Connecticut is third in Lottery expenditures, it is the highest in overall gambling expenditures, with more than three times the per capita expenditure as the second highest (Massachusetts). This state's high per capita gambling expenditure could be attributable to the presence of casinos and/or other types of gambling permitted in the state. A state's per capita gambling expenditure figure may be misleading because residents of any state can gamble there; in addition, a significant percentage of the expenditures could have come from people who are not residents of that state. Thus, the New England per capita expenditure on all gambling is probably a more accurate figure.

8. Students should divide the New England per capita expenditure on the Lottery by 365: $342.90/365 = $0.94 per day

9. Students should divide the New England per capita expenditure on the gambling by 365: $955.08/365 = $2.62 per day



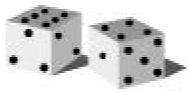
Blackline Master 2A Number Sense

Name ______________________________ **Date** ______________________

Match each category in column A with a value in column B.

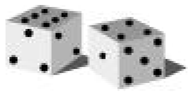
Column A	Column B
______ 1. The distance in miles from coast to coast in the contiguous United States	a. 4 million
______ 2. The number of cigarettes smoked annually in the United States	b. 400,000
______ 3. The population of the United States	c. 500 billion
______ 4. The number of people that die on Earth each day	d. 275 million
______ 5. The amount of money spent on legal gambling in the United States in one year	e. 2,840
______ 6. The number of people who die as a result of smoking each year in the United States	f. 6 billion
______ 7. The population of Louisiana	g. 5.6 billion
______ 8. The amount of money in the United States (cash and checking accounts)	h. 250,000
______ 9. The population of the world	i. 231 billion
______10. The amount of money spent on movies in the United States in one year	j. 1.1 trillion

Note: All of the numbers provided are approximations.



Answer Key Blackline Master 2A

Column A	Column B
__e__ 1. The distance in miles from coast to coast in the contiguous United States	a. 4 million
__c__ 2. The number of cigarettes smoked annually in the United States	b. 400,000
__d__ 3. The population of the United States	c. 500 billion
__h__ 4. The number of people that die on Earth each day	d. 275 million
__i__ 5. The amount of money spent on legal gambling in the United States in one year	e. 2,840
__b__ 6. The number of people who die as a result of smoking each year in the United States	f. 6 billion
__a__ 7. The population of Louisiana	g. 5.6 billion
__j__ 8. The amount of money in the United States (cash and checking accounts)	h. 250,000
__f__ 9. The population of the world	i. 231 billion
__g__ 10. The amount of money spent on movies in the United States in one year	j. 1.1 trillion



Blackline Master 2B Time Flies

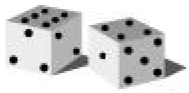
Name ______________________________Date______________________

Teenage Smoking Facts

- Statistically, each cigarette robs a regular smoker of 5.5 minutes of life.
- A teenager who smokes will smoke for an average of 25 years.
- A teenage smoker smokes about 0.6 pack per day.

Calculate each of the following. Use the result from question 1 to answer question 2; use the result from question 2 to answer question 3, and so on.

1. A pack of cigarettes has 20 cigarettes in it. How many cigarettes does a teenager smoke per day?
2. If a teenage smoker smokes ______ cigarettes a day, how many minutes of life would he or she lose per day?
3. If a teenage smoker loses ______ minutes of life per day, how many minutes of life would be lost per year?
4. If a teenager smoker loses ______ minutes per year, how many minutes would be lost in 25 years?
5. If a teenage smoker loses ______ minutes in 25 years, how many hours is that?
6. If a teenage smoker loses ______ hours in 25 years, how many days is that?
7. How many weeks of life does a teenage smoker lose who smokes for 25 years?
8. How many years of life does a teenage smoker lose who smokes for 25 years?



Answer Key Blackline Master 2B

1. $0.6 \times 20 = 12$ cigarettes/day
2. $12 \times 5.5 = 66$ minutes lost/day
3. $66 \times 365 = 24{,}090$ minutes lost/year
4. $24{,}090 \times 25 = 602{,}250$ minutes lost in 25 years
5. 602,250/60 = 10037.5 hours
6. 10037.5/24 = 418.229 days
7. 418.229/7 = 59.747 weeks
8. 59.747/52 = 1.15 years

Activity 3 Statistics in Everyday Life

Objectives

- Understand the concepts of mean, median, and mode
- Understand the concept of sampling
- Use observations about differences between samples to make conjectures about the populations involved
- Recognize misleading statistics
- Use mathematics to analyze media messages

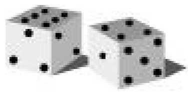
Materials	paper, pencils
Time	30–45 minutes
Math Idea	Most people use statistics and know more about statistics than they think they know. Charts of average heights and weights, batting averages, the number of hamburgers sold to date are all examples of statistics we encounter in everyday life. When you refer to information you read about health—for example, that regular exercise reduces the risk of cardiovascular disease—you are relying on statistical analyses from research studies.

Prior Understanding

Students should know how to find the mean, median, and mode for a set of values.

Introduction

You can use or adapt the following scenario as an introduction to the activity.



During the baseball strike of 1994, players' annual salaries were constantly being discussed in the media. The average baseball player's annual salary was often quoted as being $1.2 million. If you chose a player at random, would you be guaranteed that he would make $1.2 million a year?

Discussion

Discuss how useful the salary figure alone is. Ask why someone would choose to report the $1.2 million figure, which represents the mean salary, rather than the **median** salary, which was $500,000.

Knowing the $1.2 million figure alone is not particularly useful, since most people would not know which average measure (**mean**, **median**, or **mode**) was being reported. Knowing that it is the **mean** salary does not guarantee that a randomly selected player will earn that much. A few salaries could be much, much higher and many salaries could be much lower, producing the same **mean**. Someone who wanted the public to believe that baseball players are millionaires would want to report the **mean** salary; however, knowing that the median baseball player's salary was $500,000 tells you that half the players earned less than that amount and half earned more. In this case, the **median** salary is a more accurate portrayal of what the "average" baseball player earns.

Exercise 1

Tell students that you read in the newspaper that the average Harvard graduate from the class of 1990 makes $600,000 a year. Ask students what thoughts first come to mind when they hear this information. Discuss whether they should accept figures quoted in the media on faith.

Discussion

Students should recognize that they should not accept figures on faith. When students encounter statistics, some of the questions they should ask are:

- How were they obtained? Were they derived from a random sample or from a collection of anecdotes?
- How accurate are the figures? Do the figures measure what they purport to measure? Does the correlation suggest a causal relationship or is it merely a coincidence?
- How is the reporter connected to the story? Are there other ways to tally any figures presented? Is the precision of measurements meaningful?

From the Harvard salary information, most students will initially conclude that Harvard graduates make a lot of money, or that they (or their children) should go to Harvard, or something similar. The first item students should question is how representative of the class of 1990 was the sample used.

Students should want to know how the average income of Harvard graduates compares with the average incomes of graduates from other colleges. If the figures are close to being the same, then perhaps attending Harvard is not an important factor in earning potential.

Suppose the average income reported by the researchers is correct (representative). Should students immediately assume that Harvard is a better college to attend than any other college? What other information would they need?

Suppose researchers learned that the average person who graduated from college in 1990 makes $600,000, how would that change students' assumptions about Harvard?

Exercise 2

Suppose the researchers gathered data by mailing questionnaires to every member of the 1990 graduating class. Discuss some of the potential problems they might have encountered and how these problems might have affected the data they obtained. Discuss whether some of the problems could have been avoided by choosing a random **sample**.

Discussion

Discuss whether students believe that all the graduates are likely complete and return the questionnaires. What are the characteristics of the people who are likely to participate in the survey? Why might some people chose not to participate? Discuss other factors that might make affect the accuracy of this salary figure.

People who are contacted but refuse to participate in the survey could be significantly different from those who do. Students should consider factors such as who would be more likely to disclose his or her income to a stranger—the president of a successful company or an unpublished poet who works a low-paying job? Students should also consider what incentives might have been offered to get people to participate. Students should realize that the groups discussed above would bring the average annual income level down considerably if they were included in the study. In addition, it would help to know whether the researchers just asked people about their salaries or did they verify the figures somehow. The respondents' self-reported data could have been too high (because of pride) or too low (because they also lied on their income tax reports). In addition, respondents could have had many sources of income other than salary (consulting fees, investments, etc.), and simply estimated inaccurately.

The researchers may not have been able to locate all the members of the class of 1990. The people who could not be located might be significantly different from those who were located; for example, there may be a group of 1990 class members who are homeless, in jail, or are difficult to locate for some similar reason. In terms of income, these people would probably be very different from those who attended class reunions and kept in touch with the college. The same potential problems exist with a random sample. If the researchers only succeed in obtaining responses from 20% of a random sample, the results of the study may not be representative of the population being investigated.

Activity 3 Statistics in Everyday Life Teacher Support

Vocabulary

mean the arithmetic average of a set of numbers

median the middle value in a set of values that have been arranged in ascending or descending order; the midpoint, or the value in a distribution above and below which 50% of the values lie

mode the number that occurs most frequently in a set of numbers

sampling the process of selecting a group of people to be used as representative of an entire population

Ongoing Assessment

Assume that the reported annual average of $600,000 is representative of the Harvard class of 1990. If you learn that the average 50-year-old surgeon makes $800,000, and 85% of the Harvard class of 1965 are surgeons, how would this information change your interpretation of the reported figure? *(Students might then say that Harvard graduates do have high incomes, but their income level is not necessarily related to their attendance at Harvard or the year they attended.)*

Added Practice 3 Statistics in Everyday Life

Name ______________________________**Date**______________________

1. This week ask your family to help you locate everyday statistics in newspapers or magazines you have at home. Cut out at least one example of statistics used in an article. Read the article with whichever family member is helping you with this project, and try to answer the following questions:

(a) What sample is being used to describe the data? (Who did the researchers interview or survey in order to come up with their data?)

(b) How big is the sample?

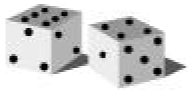
(c) Do you think this sample is representative of the overall population being discussed?

(d) What is the conclusion being drawn from the statistics?

(e) How much do you trust the information in this article?

2. *How much money do you think you need to fulfill your dreams?*

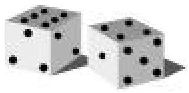
The Roper organization regularly polls Americans on this question. Every year the figures go up, in yearly leaps as great as $18,200. Last year 1,993 people were asked the question, and the *median* sum mentioned was $102,000 a year. However, the number of those requiring a million a year for their dreams had nearly doubled since the year before. Why does this example state the median sum as the average measure?



Answer Key Added Practice 3 Statistics in Everyday Life

1. Answers will vary.
2. The median is probably the most accurate measure of the "average" value.

This is true because there are a lot of people who say they would need a very large amount (for example, a million dollars). If the researchers used the mean value, these large amounts would inflate the average to a value that may not be representative of the group as a whole. For example, suppose you were surveying 10 people about the amount of money they would need to fulfill their dreams. If 7 of the 10 reported figure between $100,000 and $150,000 and the other three said that they needed a million dollars, the mean value would be somewhere around $400,000, which is not really representative of the group as a whole, or even the majority of the group. The median value, which would be around $130,000, would be a better approximation of the "average" member of the group.



Activity 4 Heads or Tails?

Objectives

- Understand concept of probability
- Understand concept of randomness
- Conduct trials and observe outcomes
- Relate theoretical probabilities to actual situations and experimental probabilities

Materials	coins, paper, pencils, calculators
Time	30–45 minutes
Math Idea	(A) When flipping a coin, the probability that the coin lands heads up is the same as the probability that the coin lands tails up—1/2 or 50%. (B) When flipping two coins, the probability that the coins show the same side (2 H or 2 T) is the same as the probability that the coins show different sides (1 H, 1 T)—1/2 or 50%.

Prior Understanding

Students should know how to convert among fractions (ratios), decimals, and percents. They should also know how to find simple theoretical probabilities.

Introduction: Gambling Connection

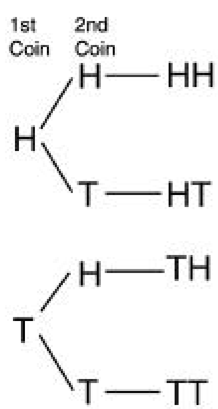
You can use or adapt the following scenario as an introduction to the problem. Then have students do the activity before revealing the answer and its explanation.

Two friends of yours are betting on the outcome of coin tosses. One of them always likes to bet on tails, and tails has come up three times in a row. She decides to change her bet for the next toss. She reasons that after three tails, a heads is “due.” What is your advice?

Discussion

(A) Have students answer the question based on their experimental results. (Although responses will vary, students should recognize that whether or not the friend changes her bet, she can't improve her chances of winning.) Stress that the **outcome** of any one trial (or a small group of trials) cannot be predicted with certainty. The **outcome** of coin tosses is always **random** and the probability for heads is the same as the probability for tails. Previous **outcomes** do not affect current or future **outcomes** in any way. No matter how many times in a row a coin lands tails up, there is still only a 50% chance that it will land heads up on the next flip.

(B) Students can also draw a **tree diagram** to determine **possible outcomes**. To help students understand that H T is a different **outcome** from T H, have them think about it as flipping two different coins, such as a dime and a penny.



Using this method, students should note that there are 4 **possible outcomes**, each with a probability of 1/4 or 25%. Using **theoretical probabilities** with two events, as is the case here, the probability that both will occur is the product of the two separate probabilities. For each coin there is a 50% chance of landing heads up, so the **theoretical probability** of both being heads (or tails) is $.50 \times .50 = .25$ or 25%. In 2 of the 4 **outcomes**, the same side is showing, so the probability is 2/4 = 1/2 = 50%. Similarly, there are 2 out of 4 **outcomes** with different sides showing.

Exercise 1

Divide students into pairs. Have partners take turns flipping a coin 20 times. Using H for heads and T for tails, to keep track of the results, have students record the number and percentage of times heads is followed by tails, and so on. Have them count the total number of heads and tails, and calculate the percent of heads and percent of tails.

$$\%\,\text{heads} = \left(\frac{\#\,\text{heads}}{20} \times 100\right) \qquad \%\,\text{tails} = \left(\frac{\#\,\text{tails}}{20} \times 100\right)$$

Exercise 2

Create a class chart on the chalkboard like the one shown. Have each pair record their results. As a class, find the total number of heads, total number of tails, and total number of flips. Then calculate the percent heads and percent tails for the total.

Pairs	Heads	Tails	% Heads	% Tails
Joe, Marie	7	13	35%	65%

Discussion

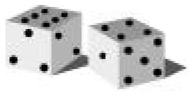
Have students compare the class percent with the percents obtained by student pairs. Discuss which results are closer to 50% and ask students to explain why. Generally, the percents obtained by student pairs will vary widely from 50% and the class percent should be closer to 50%, although some student pairs may obtain a 50% result.

Exercise 3

Without giving students coins, have them determine all the possible outcomes for flipping two coins. Ask students to calculate the theoretical probability for each outcome and the theoretical probability that both coins land with the same side up.

Discussion

Tossing a coin is a **random process**, with heads and tails being equally likely **outcomes**. With one coin, of the 2 possible **outcomes** only 1 is favorable (either heads or tails), so the **theoretical probability for each coin** is 1/2 or 50%. For two coins, there are four possible outcomes (i.e., heads/heads, tails/tails, heads/tails, tails/heads). The theoretical probability of each outcome is ¼ = .25. Students may not understand initially that heads/tails is a different outcome than tails/heads. As the number of trials increases, the **experimental probability** comes closer to the **theoretical probability**.



Activity 4 Heads or Tails? Teacher Support

Vocabulary

experimental probability probability determined by conducting a series of tests or trials and observing the number of favorable results compared to the total number of trials

outcome any possible result of an experiment or activity

probability a number from 0 to 1 that expresses the likelihood that a given event (or set of outcomes) will occur

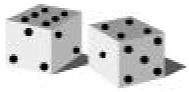
random process process in which each possible outcome has the same chance, of occurring; that is, all possible outcomes are equally likely

theoretical probability probability determined by comparing the number of ways a favorable result can happen to the total number of equally likely possible outcomes

tree diagram a diagram used to show the total number of possible outcomes

Ongoing Assessment

Let students experiment with flipping three coins and calculating the probability that all three land with the same side up. Then have them find the theoretical probability by listing all the possible outcomes. *(With three coins, there are 8 possible outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT. In 2 of the 8 outcomes, the same side is showing, so the probability is 2/8 = 1/4 = 25%.)*



Added Practice 4 Heads or Tails?

Name ______________________________Date______________________

1. List all the possible outcomes when you roll one die. Are all the outcomes equally likely? Explain.

2. Find the theoretical probability for each of the following events when rolling one die. Express your answers in fraction and percent form.

   (a) rolling a six

   (b) rolling either a four or a five

   (c) rolling an even number

   (d) rolling an odd number

3. Roll a die 30 times. Record the outcome of each roll.

   (a) Based on your results, find the theoretical probability for each outcome.

   (b) Combine your results with those of your classmates. Find the same probabilities using the totals for your class.

   (c) Compare the experimental probabilities you obtained with the theoretical probabilities you found.

4. List the possible outcomes when you roll two dice. Find the probability of rolling a 2 on both dice.

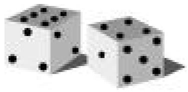
5. From a complete deck of shuffled cards, what is the probability of drawing a queen? What is the probability of drawing the queen of hearts?

Answer Key Added Practice 4 Heads or Tails?

1. Possible outcomes: 1, 2, 3, 4, 5, 6; There are six sides numbered 1 through 6; all outcomes are equally likely as long as the die is fair.
2. (a) probability of rolling a six is 1/6 or ≈ 16.67%

   (b) There are two favorable outcomes out of six possible outcomes, so the probability of rolling either a four or a five is 2/6 = 1/3 or ≈ 33.33%.

   (c) and (d) There are three even numbers on a die (2, 4, 6) and three odd numbers (1, 3, 5), so the probability of rolling an even number is 3/6 = 1/2 = 50% and the probability of rolling an odd number is 3/6 = 1/2 = 50%.
3. The probabilities obtained using the class totals are likely to be closer to the theoretical probabilities than those obtained by individual students.
4. There are 36 possible outcomes. They can be represented as ordered pairs: (1, 1), (1, 2), (1, 3), . . . , (6, 1), (6, 2), . . . , (6, 6). Students should realize that (1, 3) and (3, 1), for example, are two different outcomes. Of the 36 outcomes, only one shows 2 on both dice, so the probability is 1/36 ≈ 2.78%. Alternatively, the probability of rolling 2 on the first die is 1/6; the probability of rolling 2 on the second die is 1/6, so the probability of both happening is 1/6 × 1/6 = 1/36.
5. There are 4 queens in a deck of 52 cards, so the probability of drawing a queen is 4/52 = 1/13 ≈ 7.69%. There is only one queen of hearts, so the probability is 1/52 ≈ 1.92%.



Activity 5 Shared Birthdays

Objectives

- Understand concept of probability
- Conduct trials and observe outcomes
- Apply probability formula
- Differentiate between experimental and theoretical probabilities

Materials	pencils, paper, calculators
Time	5–10 minutes on one day to collect data
	30–40 minutes on the next day to analyze data
Math Idea	In any group of 23 randomly chosen people, there is about a 50% chance that two or more people in that group will have the same birthday.

Prior Understanding

Students should know how to convert among fractions (ratios), decimals, and percents. They should also know how to find simple theoretical probabilities using the probability formula.

Introduction: Gambling Connection

You can use or adapt the following scenario as an introduction to the problem. Then have students do the activity before revealing the answer and its explanation:

> *Sam notices there are 23 people with their dogs at the pet show. He bets his friend Rick that no two people in the group have the same birthday. Who is more likely to win the bet—Sam or Rick?*

Discussion

First have students evaluate whether Sam or Rick is more likely to win the bet, based on their experimental results. *(Students' evaluations will depend on the results they obtained.)* Then have them decide based on the theoretical probability. *(It is slightly more likely that Sam will lose and Rick will win.)*

Exercise

Have several students be data collectors in a survey of about 100 students. Have data collectors ask each student to write his or her birthday (month and day) on a slip of paper. Then have them collect the slips and bring them to class.

Pool all the slips of paper and randomly divide them into groups of 23. Divide the class into small groups, giving each group a set of 23 birthday slips. Have group members go through their slips one at a time until they get a match. Students can use a chart like the one shown to keep track of the birthdays.

Slip	Birthday
1	May 3
2	December 15
3	
4	
5	

Next, Record the number of sets that have matches. As a class determine what percent of the sets have shared birthdays.

$$\frac{\text{number of groups with matches}}{\text{total number of groups}} \times 100 = \%\text{with shared birthdays}$$

Discussion

Discuss the results with the class. If their results were not close to 50%, ask students to account for the difference. Results will vary depending on the number of students surveyed; if there are only two or three sets of 23 students, the percent is likely to be less than 50%; if there are more sets of 23 students, the percent is likely to be closer to 50%.

Be sure students understand that the activity statement expresses **a theoretical probability**, whereas their results represent an **experimental probability**. Explain that this activity is similar to tossing a fair coin. Just as there is a 50% chance of getting heads and a 50% chance of getting tails on any one toss, there is about a 50% chance of having a shared birthday and about a 50% chance of not having a shared birthday in any one group of 23 people. You can demonstrate this point by having students toss a coin once for each set of 23 birthdays. Then calculate the percent of tosses that came up heads. If there are only three tosses (three sets of 23), heads might only come up once (33.33%); if there are 10 tosses (ten sets of 23), heads might come up 4 or 6 times (40% or 66.67%); if there are 100 tosses, heads might come up 49 times (49%); and so on. The larger the number of trials (the more sets of 23), the closer the **experimental probability** will be to the **theoretical probability** of about 50/50.

To show students how to obtain the **theoretical probability**, have them consider a group of 23 people lined up in a row. Once they know the first person’s birthday, the **probability** that the second person’s birthday is different from the first’s would be 364/365 (exclusive of leap year). The **probability** that the third person’s birthday is different from both the first and second’s is 363/365. The **probability** that the fourth person’s birthday is different from the other three’s is 362/365. Suppose the pattern continues until you reach the 23rd person. The expression

$364/365 \times 363/365 \times 362/365 \times \ldots \times 343/365$ represents the probability that all 23 people have a different birthday. Let students use calculators to find the product (0.4927027656760145927745827716629 67) and change it to a percent (» 49%). Since this is the probability that *no two* people in the group have the same birthday, the probability that *at least two* people have the same birthday is » 51%.

Activity 5 Shared Birthdays Teacher Support

Vocabulary

probability a number from 0 to 1 that expresses the likelihood that a given event will occur

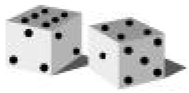
experimental probability probability determined by conducting a series of tests or trials and observing the number of favorable results compared to the total number of trials

theoretical probability probability determined by comparing the number of ways a favorable result can happen to the total number of equally likely possible outcomes

mathematical notation meaning about or approximately equal to

Ongoing Assessment

Have students speculate about the chances that two people in a group of 30 have the same birthday. Divide the birthday slips into groups of 30 and have students repeat the activity to determine an experimental probability for this situation. Then let students calculate the theoretical probability and compare it to their results. *(Students' predictions should show a higher expectation than 50%. Experimental results will vary. The theoretical probability is about 71%.)*



Added Practice 5 Shared Birthdays

Name ______________________________ **Date**______________________

1. In a group of 40 people, how likely is it that two or more people have the same birthday? Make a prediction, then complete the activity. You may wish to use a separate sheet of paper.

   (a) Make a list of 40 people whose birthdays you can look up in an encyclopedia or other reference book. You might choose presidents; astronauts; sports figures; movie, television, or music personalities; explorers; scientists; and so on. Look up and record their birthdays. When you get a match, you can stop; or you can continue to see whether more than two people on your list share a birthday.

   (b) Combine your results with those of your classmates to calculate the percent of the time there are shared birthdays in a group of 40 people.

   $$\frac{\text{number of groups with matches}}{\text{total number of groups}} \times 100 = \text{\% with shared birthdays}$$

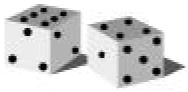
   (c) Use a calculator to determine the theoretical probability that two or more people in a group of 40 share a birthday. Compare this result to the experimental probability you obtained.

2. Within any group of 367 people, what is the likelihood that at least two will have the same birthday? Explain your answer.

3. In a group of 20 telephone numbers selected at random, what is the likelihood that two or more numbers match in the last two digits?

Answer Key Added Practice 5 Shared Birthdays

1. Students' predictions should indicate an expectation higher than 71% based on their previous experiences with this problem. The theoretical likelihood for a group of 40 people is about 89%.
2. 100%; There are 366 possible birthdays (365 days plus February 29 during a leap year), so at least two people must share a birthday. A good analogy is placing 367 letters in 366 mailboxes. Each mailbox gets one letter and one mailbox must get two letters.
3. Once the first telephone number is picked, the probability that the last two digits of second phone number **do not** match the first is 99/100; the probability that the last two digits of the third phone number do not match either the first or second is 98/100; and so on. Then $99/100 \times 98/100 \times 97/100 \times \ldots \times 81/100 \approx 0.15902 \approx 16\%$ represents the probability that the last two digits of all 20 phone numbers do not match. That means there is about an 84% chance that the last two digits will match.



Activity 6 Becoming a Legend

Objectives

- Understand concept of probability
- Use case studies to understand real-life situations involving experimental probability
- Differentiate between risks based on skill and risks based on chance
- Compute combined probability

Materials	paper, pencils, calculators
Time	30 minutes
Math Idea	In the history of baseball, a season batting average of .400 has been one of the major achievements for a hitter to attain. Since 1941, Ted Williams is the only player to have hit .400.

Prior Understanding

Students should know how to convert among fractions, decimals, and percents. They should also know how to multiply and divide fractions and decimals.

Introduction

Use the following story as an introduction to the problem. Then have students do the activity.

In 1941, Ted Williams, the legendary Red Sox slugger, was approaching the end of the season with a batting average just over .400. Williams risked falling below .400 if he played in both of the last two games. The coach offered to let Williams sit these games out. "I want to hit over .400," Williams said, according to Sports Illustrated. *"The record's no good unless it's made in all the games." Do you think Williams was taking a big risk by playing in all of the games?*

Discussion

Students' opinions will vary. Some may argue that because of his great skill and experience it was likely that he'd keep up his average. This is different from risks based solely on chance, like a coin toss.

Exercise 1

In baseball, batting average for a season is calculated by dividing the player's number of hits during the season by the number of times that the player has been at bat. By the end of the third from last game, Williams had 179 hits out of 448 times at bat. Have students calculate his batting average and use it to solve these problems.

- Find the **probability** that Williams would get 2 hits in two consecutive times at bat.
- Find how many hits Williams would be expected to get out of 8 times at bat (the number of times he could be up during a double-header).

Discussion

Students should realize that a batting average is an **experimental probability**—it is based on previous occurrences, and as such, is only an estimate. There is no guarantee that future trials will follow the same pattern because the outcomes are not based only on chance but are also influenced by a player's skill. The greater the number of trials, the better the estimate. A player who gets 4 hits out of 10 times at bat has the same batting average as one who gets 40 hits out of 100 times at bat. However, the latter player's average is a better estimate of his or her ability. Williams' average of 179/448 = 0.39955 was a good indicator of his ability. Because batting averages are carried out to only three places, this batting average would round to.400.

Exercise 2

(a) Assuming Williams got that many hits on the last day of the season, have students determine what his final batting average would have been.

(b) Assuming Williams had 8 times at bat, have students determine how many hits he would have needed to finish with a .400 batting average or better.

(c) In the last two games, Williams got 6 hits out of 8 times at bat. Have students calculate his final batting average.

Discussion

The **combined probability** that Williams would get 2 hits in a row can be estimated by finding the product of the separate **probabilities**, or (.39955)(.39955) = .1596 or about 16%. In actuality, if he got a hit the first time, his batting average would go up slightly, but not enough to make a difference in the *estimate.*

Using his .39955 batting average, he would be expected to get $.39955 \times 8 \approx 3$ hits. (Students should solve the equation: $\frac{x}{8} = .39955$.) With 3 more hits, his average would have been 182/456 = .39912 or .399 (it would have gone down). With 4 hits, his average would have been 183/456 = .401, so he would have needed 4 hits out of 8 times at bat. His final average was

185/456 = .406.

Discuss with students whether they think it was wise for Williams to play, and what they would do if they were in a similar position.

Activity 6 Becoming a Legend Teacher Support

Vocabulary

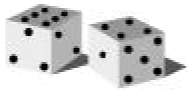
probability a number from 0 to 1 that expresses the likelihood that a given event (or set of outcomes) will occur

experimental probability probability determined by conducting a series of tests or trials and observing the number of favorable results compared to the total number of trials

combined probability the probability that two or more events will all occur

Ongoing Assessment

Out of 1,000 consecutive foul shots, a basketball player makes 698 of them. What is the probability that he or she will make two foul shots in a row? *(.698* $\times$ *.698 = .487 or about 49%)* How many more shots must he or she make in a row to have a .700 average? *(7; students can solve the equation:* $\frac{698+x}{1000+x}=.700$ *)*



Added Practice 6 Becoming a Legend

Name ______________________________**Date**______________________

1. Your favorite basketball team is down by one point. Time has run out in the fourth quarter, and your team has a player at the foul line for two shots. You know that the player has made 80% of the foul shots he has taken this season. What is the probability that your team will win the game?

2. On Friday night, the weather forecaster says that there is a 99% chance of rain for Saturday. Saturday turns out to be a beautiful sunny day. Can you say that the weather fore-caster's prediction was wrong? Why or why not?

3. Auto insurance companies base their rates according to the probability that a driver will have an accident during the course of a year. The probabilities that drivers of different ages will be involved in a crash are given in the table be-low.

Age group	Probability of being involved in a crash
Under 20	.153
21–24	.102
25–34	.073
35–44	.057
45–69	.046
Over 69	.040

According to the table, what is the probability that a driver under the age of 20 will be involved in a crash in two consecutive years? What is the probability that a driver between the ages of 45 and 69 will be involved in a crash in two consecutive years? Which driver do you think the insurance company will charge higher rates? Why?

Answer Key Added Practice 6 Becoming a Legend

1. The probability that he will make the first shot is 80%, and the probability that he will make the second shot is 80%, so the probability that he will make both is .80 x .80 = .64. Your team has a 64% chance of winning.
2. You can't necessarily say that the weather forecaster is wrong. Weather predictions are based on experimental probabilities. With a 99% chance of rain, there is still a 1% chance of sun. No matter how probable rain is, it is still possible to get sun. In other words, a probability only tells you the likelihood of an outcome, not exactly what the outcome is going to be.
3. (.153)(.153) = .023 or 2.3%

   (.046)(.046) = .002 or 0.2%

   Insurance company would charge a driver under 20 higher rates because that driver is more likely to be involved in a crash that a driver between the ages of 45 and 69.

Activity 7 You Bet Your Life!

Objectives

- Understand concept of probability
- Conduct trials and observe outcomes
- Use a mathematical model to approximate a real-life situation
- Understand applications of probability in real-life situations

Materials	paper, pencils, calculators
Time	45 minutes
Math Idea	Events in real life are not solely determined by chance; nor are all outcomes equally probable. Even so, probability can be used as a basis for evaluating risks and making real-life choices.

Prior Understanding

Students should know how to convert among fractions, decimals, and percents.

Introduction

Use the following information as an introduction. Then have students do the activity.

- *The American Cancer Society estimates that each year more than 430,000 people in the U.S. die as a result of smoking.*
- *Smoking accounts for nearly 90% of deaths from lung cancer.*
- *The risks of dying from lung cancer are 23 times greater for male smokers and 13 times greater for female smokers than for non-smokers.*
- *In the U.S., more than 70% of adults who smoke began smoking before age 18.*

Would you bet your life by smoking?

Discussion

Students' opinions will vary. The model used assumes that each student will get, at most, one disease, which is a simplified and conservative estimate of risk for smokers. It is important for students to understand that there are varying probabilities for contracting different diseases, and that some diseases are more likely to be contracted than others. It is up to them to evaluate the risks.

Exercise 1

Ask students to name other diseases related to smoking. Make a list on the chalkboard. Ask students whether they think every smoker will develop at least one of these diseases.

Divide the class into small groups (4 or 5 students each). Give each group a pair of dice. Have each group designate one student to be the record-keeper. Give a copy BLM 7 to each record-keeper. Explain that that each member of each group will take a turn rolling the dice. Each student in the group will roll the dice one time while the record-keeper enters the outcome and its corresponding disease in each row of the table. As the record-keeper records the outcomes, he or she should inform the dice-roller whether he or she will "contract a disease" and, if so, which one.

When all the groups have finished, make a class tally on the chalkboard. You can use a chart like the one shown to summarize the results.

Discuss the results and have students evaluate the risks.

Disease	Group Total	Group Total ¸ Total Number of Students	Percent of Class “With Disease"
H	2	2/20	10%
CL	3	3/20	15%
CM	1	1/20	5%
ST			
EM			
LC			
BR			
HD			
UL			
WR			
CT			

Discussion

Smoking-related diseases include cancer of the larynx, pharynx, mouth, lips, tongue, esophagus, bladder, kidney, stomach, pancreas, and colon; cardiovascular disease including stroke, heart attack, peripheral vascular disease, aneurysm, and atherosclerosis; pulmonary illnesses such as pneumonia, emphysema, and bronchitis; reproductive problems such as stillbirth, low birth weight, and reduction of fertility; other effects such as cataracts, gum disease, ulcers, wrinkles, hypertension, and delayed healing. Diseases from each of the categories are represented in the key for BLM 7.

Students should realize that some smokers will get one of these diseases, some will get more than one of these diseases, and some smokers will get none. Students may have heard stories about people who smoked, drank, and "had a good time" all their lives and are still healthy and happy at age 100. As it is rare to hear of an adolescent with lung cancer, an adolescent who is currently healthy tends to think long-term health consequences are too far off in the future to be real or threatening. It is true that there are individuals who engage in behaviors that have a high **probability** of leading to disease, but who "beat the odds." The examples used here are for simulation purposes. The actual incidences of these diseases are influenced by many factors.

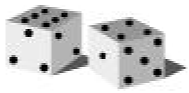
Activity 7 You Bet Your Life! Teacher Support

Vocabulary

probability a number from 0 to 1 that expresses the likelihood that a given event (or set of outcomes) will occur

Ongoing Assessment

In the model, why do you think lung cancer corresponded to a total of 7, while healthy corresponded to a total of 2? *(The probability of contracting lung cancer is higher than the probability of contracting any of the other diseases; the probability of staying healthy is very small. When rolling two dice, there is only one way to roll a sum of two for a probability of 1/36 or 2.6%; there are six ways to roll a sum of seven for a probability of 6/36 or 16.7%. This way the assignments better reflect the real-life situation.)*



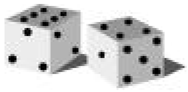
Added Practice 7 You Bet Your Life!

Name ______________________________ **Date** ______________________

Suppose that recently you have been diagnosed with a minor physical ailment. You are considering having your doctor treat your condition with a medical procedure, but you are not sure how safe this procedure is. To determine whether the chance of treatment is worth the risk of complications, you make an appointment to speak with your doctor about it. “Don't worry," your doctor tells you, "this procedure is 99% safe." You mention that you had heard that there can be complications with this procedure, and you tell that to your doctor. "Complications are a possibility," your doctor replies, "but the risk is only one-in-a-million." You ask your doctor how often patients in the hospital where he works have had problems with the procedure. "Oh, it usually goes quite well," he answers. What is the problem with your doctor's assessment of this procedure's risk?

Answer Key Added Practice 7 You Bet Your Life!

Your doctor has reported three significantly different levels of risk. The first time, the doctor says that the procedure is 99% safe, meaning 99 times out of 100 there would be no complications. This is the same as saying 1 person out of 100 or 10,000 out of a million persons *will* have complications. The second time, the doctor says that the procedure has a one-in-a-million risk, which means that 1 person out of a million will have complications—that makes your chances of risk-free procedure a lot better than the 10,000 out of a million risk he reported the first time. The third time, the doctor says that the procedure usually goes well, which does not give you any specific data on which to base your decision. At the least, you can assume "usually" means that the procedure goes well more often than it does not go well—in other words, that it goes well at least 51% of the time and goes badly at most 49% of the time. At this level of risk, as many as 490,000 people out of a million will have complications. Clearly, this doctor does not understand probability—he does not know whether 1, 10,000 or 490,000 people out of a million will have problems with this procedure.



Blackline Master 7 You Bet Your Life!

Name ______________________________Date______________________

Use the table and the key below to record the results for your group.

Student	Dice Total	Disease Letter Code
#1		
#2		
#3		
#4		
#5		

Key

Dice Total	Disease	Letter Code
2	Healthy (no disease)	H
3	Cancer of the lips	CL
4	Cancer of the mouth	CM
5	Stroke	ST
6	Emphysema	EM
7	Lung cancer	LC
8	Bronchitis	BR
9	Heart disease	HD
10	Ulcers	UL
11	Wrinkles	WR
12	Cancer of the tongue	CT

Activity 8 The Gambler's Fallacy

Objectives

- Apply the probability formula
- Compute combined probability
- Define *dependent* and *independent* events
- Determine the probability of two or more events
- Define the *gambler's fallacy* and relate it to mathematical probability

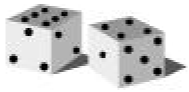
Materials	paper, pencils, calculators
Time	30 minutes
Math Idea	The belief that because a certain event has *not* occurred many times in a row that is more likely to occur the next time is known as the *gambler's fallacy.*

Prior Understanding

Students should know how to convert among fractions, decimals, and percents. They should also know how to find simple theoretical probabilities using the probability formula.

Introduction: Gambling Connection

You can use or adapt the following scenario as an introduction to the problem. Then have students do the activity and discuss the solution.



Sasha has three younger brothers, and her mother is about to give birth again. She is betting her brothers that this time her mother will have a girl. Assuming the chances of being born a boy and the chances of being born a girl are the same, how certain is Sasha of winning her bet?

Discussion

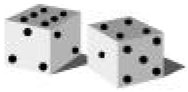
Students should realize that Sasha is a victim of the *gambler's fallacy.* She is confusing a single event with a sequence of four events and is mistaking **independent events** for **dependent events.** An outcome of boys on the three previous births does not remove "boy" from the pool of possible outcomes for the next birth, nor does it guarantee "girl" as the only possibility. No matter what has happened before, the probability of her mother giving birth to a girl (or a boy) at any one time remains 50%.

Exercise 1

Have students list the possible outcomes for tossing a coin four times. Then have them determine the probability of getting four tails in a row.

Have students list the possible outcomes for tossing a coin the fourth time after three tails have already come up. Then have them determine the probability that tails will come up again.

Have students compare these two problems, identifying any similarities and differences.



Discussion

When determining the **probability** of two or more events, it is important to know (a) whether you are determining the **probability** of a single event or the **probability** of a group of events, and (b) whether the outcomes of the events are **independent** or **dependent**.

The tossing of four coins in a row consists of *four events* with a total of 16 possible outcomes: HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, TTTH, TTTH, TTHT, TTHH, THTT, THTH, THHT, THHH. Only one of these outcomes shows four tails, so the **probability** of getting four tails is 1/16. The four tosses are ***independent***—the outcome of the first toss does not affect the outcome of the second, third, or fourth in any way. On each toss, the **probability** of getting tails (T) is 1/2, so the combined **probability** of getting four tails is $1/2 \times 1/2 \times 1/2 \times 1/2 = 1/16$ or 6.25%.

If three tails have come up, the tossing of the next coin is a ***single event*** whose outcome is ***independent*** of what has already happened. The **probability** of getting tails again is still 1/2 or 50%.

Here it does not matter what the **probability** of getting four heads in a row is: at this point, we are only dealing with the *next* toss. Students should note that although determining the **probability** of an outcome for a *group* of coin tosses is different from determining the **probability** of an outcome for *one* coin toss, the individual outcomes are **independent**.

Exercise 2

Have students determine the probability of drawing two aces in a row from a well-shuffled, complete deck of cards, if the drawn cards are replaced and the deck is shuffled again after each draw.

Have students determine the probability of drawing two aces in a row from a well-shuffled, complete deck of cards, if the drawn cards are not replaced and the deck is shuffled again after each draw.

Have students compare these 2 problems, identifying any similarities and differences.

Discussion

The drawing of two aces in a row consists of two events. When the drawn cards are returned to the deck, the two events are **independent events**. Each time there are 4 aces out of 52 cards to choose from, so the **combined probability** is $4/52 \times 4/52 = (1/13)^2 = 1/169$ or about 0.59%. Students should note that it is possible to draw the same ace twice.

When the drawn cards are *not* returned to the deck, the two events are ***dependent events***— the result of the first draw affects the potential outcome of the second draw. If the first draw is an ace, there are 3 aces left out of 51 cards in the deck. The **probability** of drawing another ace is now 3/51 and the **combined probability** is $1/13 \times 3/51 = 1/221$ or about 0.45%.

Activity 8 The Gambler’s Fallacy Teacher Support

Vocabulary

probability a number from 0 to 1 that expresses the likelihood that a given event (or set of outcomes) will occur

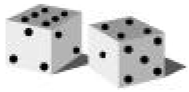
combined probability the probability that two or more events will all occur

independent events events in which the outcome of the first event *does not* affect the outcome of subsequent events

dependent events events in which the outcome of the first event *does* affect the outcome of subsequent events

Ongoing Assessment

What is the probability of drawing four clubs in a row from a well-shuffled, complete deck of cards if the drawn cards are not replaced? *(13/52 ´ 12/51 ´ 11/50 ´ 10/49 = 0.00264)*



Added Practice 8 The Gambler's Fallacy

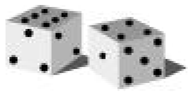
Name ______________________________Date______________________

1. Suppose you have already drawn two cards from a well-shuffled, complete deck and neither one is a five. What is the probability that you will draw a five the next time if the first two cards are not replaced?

2. Suppose you have already drawn a four and a five from a well-shuffled, complete deck. What are the chances of drawing a five the next time if the first two cards are not replaced?

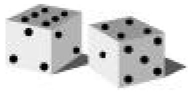
3. A friend of yours has a cousin who was in a minor plane crash and survived. Whenever your friend has to fly in an airplane, she insists on being accompanied by her cousin, figuring that the probability of her cousin being in two plane crashes is very small. Is your friend's reasoning correct? Why or why not?

4. An acquaintance of yours is concerned about a recent terrorist bombing of an airline flight. He decides that on all future flights he takes, he will bring a bomb in his suitcase. He reasons that the probability of two bombs being on a plane is very, very low. Will his plan decrease his chances of being killed by a terrorist bombing while flying? Why or why not?



Answer Key Added Practice 8 The Gambler's Fallacy

1. On the next draw there are 50 cards left in the deck, 4 of which are fives. The probability is 4/50 = 8%.
2. On the next draw there are 50 cards left in the deck, only 3 of which are fives. The probability is 3/50 = 6%.
3. Your friend's reasoning is not correct. Plane flights are not dependent events—the outcome of one flight does not have an effect on the outcome of subsequent flights. Assuming that the probability of a commercial flight crashing is one in a million, a person will be subjected to that one-in-a-million risk each time he or she takes a commercial flight, whether or not he or she has been in a previous crash. In reality, there are specific causes of plane crashes: faulty or damaged airplane parts, engine failures, weather conditions, pilot incompetence, etc. However, because you cannot evaluate all of the variables each time you take a flight, you have to assume that plane crashes are more or less random occurrences. That is, because the factors leading to a crash are unknown and unpredictable before the crash, you have to approach crashes as rare chance outcomes. Clearly, the presence or absence of someone who has been in a previous crash does not influence the outcome of the flight in any way.
4. Your friend's plan will not change his chances of being blown up by a terrorist. A terrorist choosing to blow up the plane that your friend is on is a random occurrence. Let's say, hypothetically, that the chance of this happening on any particular plane is 1/5,000,000. If your friend knows that he is carrying a bomb on a particular flight, then he can be 100% confident that there is at least one bomb on his plane. The combined probability of his bomb *and* a terrorist's being on his plane is the product of the individual probabilities, that is 1 x 1/5,000,000 = 1/5,000,000. Although it is true that the probability of two people putting a bomb on the same plane independently is very, very small, what your friend is carrying in his suitcase has no effect on the situation.



Activity 9 Winning and Losing the Lottery

Objectives

- Apply the probability formula
- Define *lottery*
- Use basic counting processes to find permutations or combinations in a given situation
- Determine the probability of possible combinations
- Compare the probability of winning a lottery with the probabilities of other events

Materials	paper, pencils, calculators, 250 push pins or thumbtacks, wall map of United States
Time	45 minutes
Math Idea	It is difficult to find an event that is less likely to occur than winning the lottery.

Prior Understanding

Students should know how to convert among fractions, decimals, and percents, as well as find probabilities using the probability formula. They should also know how to use the fundamental principle of counting, understand factorial notation, and differentiate between permutations and combinations.

Introduction: Gambling Connection

You can use or adapt the following as an introduction to the problem. Make a list of students' responses on the board. After students complete the activity, look back at the list and discuss any changes in students' responses.

Most state lotteries use some variation of the following procedure: players pick six numbers out of a pool of numbers such as 1 through 48 inclusively. On the scheduled day, a lottery official randomly draws six numbers from the same pool. If any player matches the six numbers drawn randomly, that player wins the jackpot. If more than one player matches the six random numbers, then the winners share the jackpot. If no one matches all six random numbers, the jackpot "rolls over" and is increased for the next drawing. What are the chances of winning and losing a lottery?

Exercise 1

Distribute copies of BLM 9 to students. Have them guess at the order of likelihood in which the given events might occur. Assign individuals or groups to research the likelihood of each event except winning the lottery.

Discussion

The events, in order from most likely to least likely, are as follows (all probabilities, except for the **lottery**, are average annual rates based on actual mortality rates):

Being killed in a car accident	one in 5,300
Being a drowning victim	one in 20,000
Choking to death	one in 68,000
Being killed in a bicycle accident	one in 75,000
Being killed by a terrorist in a foreign country	one in 1.6 million
Being killed by lightning	one in 2 million
Dying from a bee sting	one in 6 million
Winning the lottery	one in 12.3 million

Exercise 2

Have students find the total number of different ways to pick three out of four numbers: 1, 2, 3, 4. Then have them find the probability that any one combination of three randomly drawn numbers will be chosen.

Discussion

Students can use a tree diagram to find the 24 possible ways of choosing three out of four numbers, of which only four are distinct from the others (1, 2, 3; 1, 2, 4; 1, 3, 4; 2, 3, 4). This is the same as calculating the **combination** C(4, 3) = $\frac{4!}{3!1!} = \frac{4\times3\times2\times1}{(3\times2\times1)(1)} = 4$. Each **combination** of three numbers has a 1/4 or 25% possibility of being drawn.

Exercise 3

> Have students start with the numbers from 1 to 7 and find the probability of winning by choosing six out of seven numbers in a random draw.

Discussion

There are seven ways to choose six out of seven numbers (1, 2, 3, 4, 5, 6; 1, 2, 3, 4, 5, 7; 1, 2, 3, 4, 6, 7; 1, 2, 3, 5, 6, 7; 1, 3, 4, 5, 6, 7; 1, 2, 4, 5, 6, 7; 2, 3, 4, 5, 6, 7); and a 1/7 or 14.3% probability of any one of the **combinations** being drawn.

Exercise 4

> Have students calculate the probability of choosing a winning combination with:
>
> (a) 6 out of 10 numbers (1 to 10)
>
> (b) 6 out of 20 numbers (1 to 20)
>
> (c) 6 out of 48 numbers (1 to 48)
>
> Using the result obtained for 6 out of 48 numbers, have students assume that that many tickets are sold. Discuss whether or not a winner would be guaranteed.
>
> If all possible combinations of 6 out of 48 numbers are selected and no one buys more than one ticket, have students determine the number of people that would *lose* the lottery drawing.

Discussion

For 6 out of 10 numbers, students should calculate C(10, 6) = $\frac{10!}{6!4!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$ **combinations** with a 1 out of 210 (or 0.47%) chance of winning. For 6 out of 20 numbers, students should calculate C(20, 6) = $\frac{20!}{6!14!} = 38{,}760$ **combinations** with a 1 out of 38,760 (or 0.0025%) chance of winning.

For 6 out of 48 numbers, C(48, 6) = $\frac{48!}{6!42!} = 12{,}271{,}512$ **combinations** with a 1 out of 12,271,512 (or 0.0000081%) chance of winning.

To put this figure into perspective, assume there are about 4 million people in Louisiana. Suppose Louisiana had a **lottery** game that drew 6 numbers out of a pool of 48. Even if every person in the state bought 3 tickets each, there would be no guarantee that any of the 4 million people would win the **lottery**, because 12 million tickets would not cover all of the 12.3 million possible **combinations** of six numbers. Furthermore, even if 12.3 million tickets were sold, there still would be no guarantee that anyone would win, since the same losing **combination** of numbers could have been selected by more than one person. In this case, all of the 12.3 million possible number **combinations** would not have been covered, and the winning number could be one of the **combinations** not purchased.

Students should also note that none of the other events listed on BLM 9 comes close to the tiny probability of winning a state **lottery**. Nevertheless, many people probably consider winning the **lottery** much more likely than, for example, dying in a car crash. And the lottery never publicizes the fact that a person is almost six times more likely to be killed by lightning than to win the **lottery**.

Students should realize that there would be 12,271,511 losers. To give students an idea of how big a number this is, have them assume a 1-to-1 correspondence between the number of losers and number of middle school students in the United States. Ask them to find the number of

(a) classrooms full of losers (assume 30 students per class) *(12,217,511/30 = 409,050 classrooms)*

(b) the number of schools full of losers (assume 40 classes per school) *(409,050/40 = 10,226 schools)*

(c) the number of schools in every state completely full of losers *(10,266/50 = 205 schools per state).*

Using a map of the United States, have students try to put 205 push pins in just one state—the state will be full of losers. Remind the students that each push pin represents a school of 1,200 students, just like their school, and that every state in the country would have 205 push pins in it just from a single **lottery** drawing!

Activity 9 Winning and Losing the Lottery Teacher Support

Vocabulary

combination a selection of things in which order does not matter

fundamental principle of counting the number of ways of making several successive decisions is the product of the number of choices that can be made in each decision

lottery gambling game in which players buy tickets bearing combinations of numbers that must match a combination of numbers selected at random in order to win

permutation an arrangement of things in a definite order

probability a number from 0 to 1 that expresses the likelihood that a given event (or set of outcomes) will occur

Ongoing Assessment

Have students calculate the chance of winning a pick 6 out of 49 lottery. *(1 out of 13,983,816 or about 1 in 14 million)*

Added Practice 9 Winning and Losing the Lottery

Name ______________________________ **Date** ______________________

The media always focus on the single lottery winner but neglect to mention the vast number of losers. So, imagine you had a 3" × 4" photograph of every *loser* in a single lottery drawing.

1. Suppose you build a scaffolding box around the Statue of Liberty and want to cover the surface of the box with pictures of losers. The dimensions are given below. Calculate the surface area of the box (assume the box has no top or bottom).

Statue: height—151 feet

Base: height—154 feet

width—154 feet

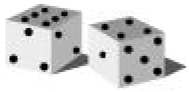
depth—154 feet

2. Determine how many photos are needed to cover one square foot of surface area and the number of photos needed to cover the entire scaffolding.

3. Determine how many Statues of Liberty you could cover with the 12,271,511 photos of losers.





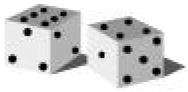



Answer Key Added Practice 9 Winning and Losing the Lottery

1. Surface Area = (height of Statue + height of base) $\times$ (width of base) $\times$ 4 = (151' + 154') $\times$ 154' $\times$ 4 = 187,880 sq. ft.
2. The area of each photo is (1/4’) $\times$ (1/3’) = 1/12 sq. ft. So 12 photos cover 1 square foot of surface area.

   187,880 sq. ft x 12 photos per sq. ft = 2,254,560 photos needed to cover the entire scaffolding
3. $\frac{12{,}271{,}511 \text{ photos of losers}}{2{,}254{,}560 \text{ photos per Statue}} = 5.44$ Statues of Liberty

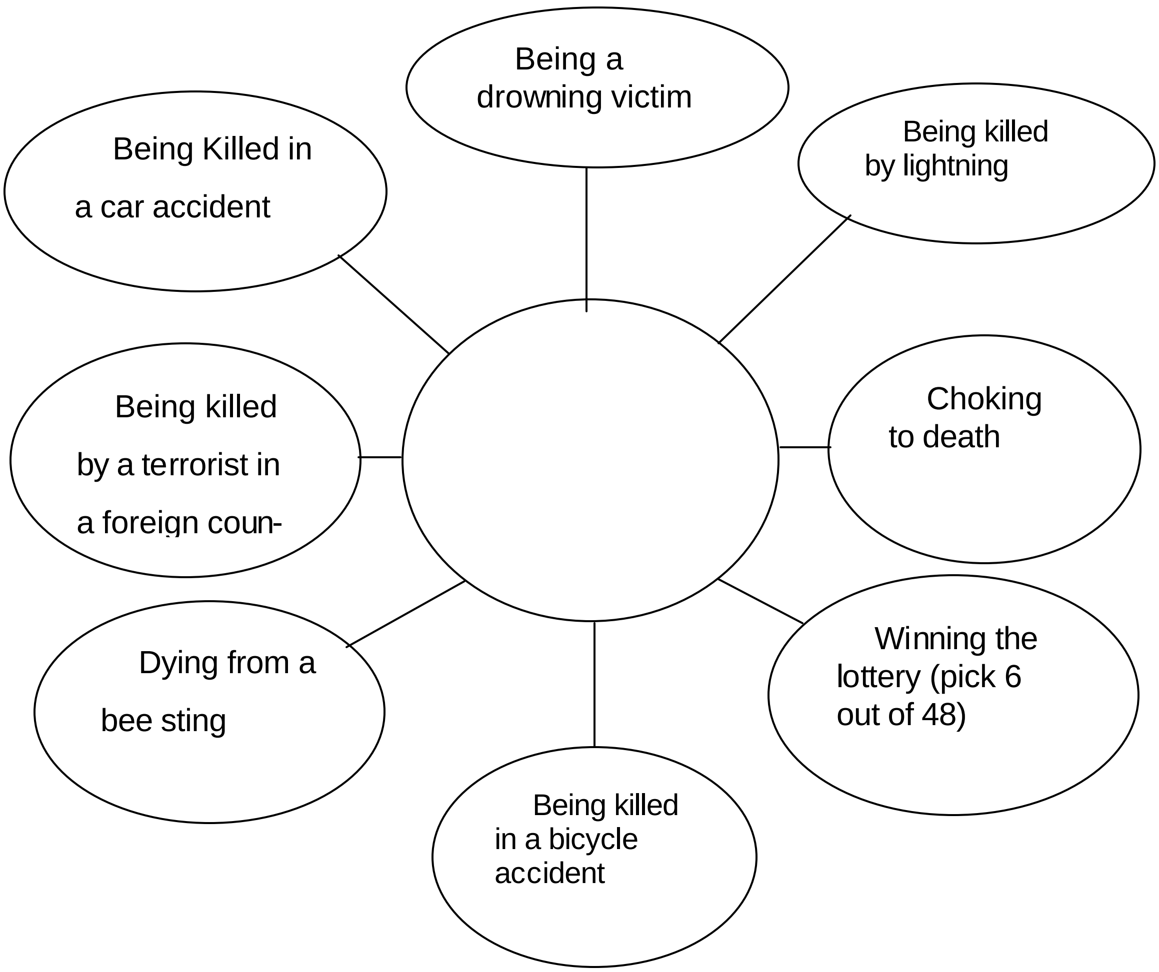
   Can you imagine seeing five of those statues completely covered with pictures of lottery losers from just one drawing-- with enough pictures of losers left over to cover the sixth statue up to its waist?



Blackline Master 9 Winning and Losing the Lottery

Name ______________________________Date______________________

Based on your knowledge or opinions, put the following events in order from most likely to least likely.



Activity 10 Red and Black Marbles and the Lottery

Objectives

- Apply the probability formula
- Differentiate between *dependent* and *independent* events
- Explain the effect of increasing or decreasing the size of a pool of numbers on chance

Materials	paper, pencils
Time	20–30 minutes
Math Idea	Once a specific combination of numbers is chosen, the composition of various groups within a pool of numbers becomes irrelevant.

Prior Understanding

Students should know how to convert among fractions, decimals, and percents, as well as find probabilities using the probability formula.

Introduction: Gambling Connection

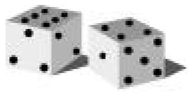
You may wish to read aloud the following to begin a classroom discussion and introduce this activity.

A lottery player might reason that because there are more spread-out combinations (such as 3, 16, 24, 30, 37, 42) than strings of consecutive numbers (such as 34, 35, 36, 37, 38, 39) or numbers that are below a certain value (such as 1, 3, 4, 6, 8, 9), he or she can increase the chances of winning by selecting numbers that are spread out over the entire range of possible numbers.

Discussion

While it is true that there are more collections of numbers spread out than there are collections of numbers under ten or collections of consecutive numbers, that is irrelevant as far as random selection is concerned. The prize is not given for matching the *pattern* of the winning numbers.

Once all the items in the pool are viewed as individual choices (as opposed to members of a group), it becomes clear that each choice has the same chance of being selected as any other choice. Although people tend to group their number choices together according to some characteristic that they think the numbers have in common, when it comes to random selection, none of the number combinations has anything in common with any other number combination: each one is an individual combination out of millions of combinations.



Example 1

Give students the following three scenarios and have them determine the corresponding probabilities.

1. A bag that contains five red marbles and five black marbles; one of the red marbles is marked with an **X**. You reach into the bag without looking and pull out a marble at random. What is the probability of choosing a black marble? What is the probability of choosing the marble with the **X** on it?
2. You remove from the bag all of the red marbles except the one marked with the **X** and put in four more black marbles. What is the probability of choosing a black marble? What is the probability of choosing the marble with the **X** on it?
3. You remove all but one of the black marbles and put in 8 more red marbles. What is the probability of choosing a black marble? What is the probability of choosing the one with the **X** on it?

Discuss the effect of choosing different collections of numbers on winning the lottery.

Discussion

For 5 red and 5 black marbles, P(black marble) = 5/10 = 1/2 or 50%; P(marble **X**) = 1/10 or 10%. For 1 red and 9 black marbles, P(black marble) = 9/10 or 90%; P(marble **X**) = 1/10 or 10%. For 9 red and 1 black marble, P(black marble) = 1/10 or 10%; P(marble **X**) = 1/10 or 10%. Students should notice that the **probability** of choosing the red marble with the **X** is the same regardless of the compositions of the remaining red and black marbles in the bag. That is, the **probability** of picking a *specific* red marble (which is different from picking *any* red marble) is the same whether 90% of the marbles are red or only 10% of the marbles are red.

Activity 10 Red and Black Marbles and the Lottery

Teacher Support

Vocabulary

probability a number from 0 to 1 that expresses the likelihood that a given event (or set of outcomes) will occur

independent events events in which the outcome of the first event *does not* affect the outcome of subsequent events

dependent events events in which the outcome of the first event *does* affect the outcome of subsequent events

Ongoing Assessment

Angela always picks the numbers 4, 8, 15, 17, 23, and 26 for the lottery. While she is away on vacation, her combination is picked. Upon returning, she decides to change her six number combination since it has already been picked. Angela believes that her number combination is less likely to be picked because it was picked only a few days ago. Is she correct? Why or why not? *(In reality this is not a bet that two lottery drawings in a row will both be the given numbers. It is only a bet that the* next *drawing will be the given numbers. Like the coin tosses discussed in the* Head or Tails? *and* The Gambler's Fallacy *Activities, earlier lottery drawings are independent events: the probability of any one outcome is always the same, regardless of what has happened before. The probability of the same numbers winning given that they won the last time, is 1 ´ 1/12,271,512 = 1/12,271,512.) [See page 98 for additional discussion]*

Added Practice 10 Red and Black Marbles and the Lottery

Name ______________________________Date______________________

1. Michael plans to use birth dates to select his lottery numbers. Then he remembers that birth dates never go past 31, and the lottery uses numbers 1 through 48. Will Michael hurt his chances of winning if he decides to use birth dates? Explain your answer.

1	2	3	4	5	6	7
8	9	1 0	1 1	1 2	1 3	1 4
1 5	1 6	1 7	1 8	1 9	2 0	2 1
2 2	2 3	2 4	2 5	2 6	2 7	2 8
2 9	3 0	3 1	?	?	?	?
?	?	?	?	?	?	?
?	?	?	?	?	?	?

2. Erin wants to pick the numbers 1, 2, 3, 4, 5 and 6, but her friend tells her that this is not a good idea because the probability of six consecutive numbers being drawn is so small. Would Erin improve her chance of winning if she chose a more varied six-number combination? Explain your answer.

3. John's sister wins the lottery. She advises him to stop playing the lottery for awhile. She reasons that his chances of winning the lottery have decreased, since the likelihood that two people in the same family would win the lottery is so small. Is she correct? Why or why not?

4. Read the advertisement, then answer each question.

WIN THE LOTTERY IN 18 DAYS OR LESS GUARANTEED!!!
STEVE PLAYERS LOTTERY POWER WORKOUT
There are over 20,000 copies of this book in print, and for a very good reason:
THIS SYSTEM WORKS!!!
It will only cost you $5.00 each day to buy the tickets and our guarantee says the rest: Either you win your state's lottery game in the first 18 days that you use the system or we will gladly refund the full purchase price of the system.
NO FINE PRINT!!!
DON'T WAIT - ORDER TODAY - ONLY $35. - / CATALOG $2.00

Name: ______________________ Address: ______________

City: ___________________ St: ______________ Zip: _____

(a) Although the advertisement states that there is "no fine print," can you think of what the "catch" might be?

(b) How much will it cost you if you buy the tickets for the full 18 days?

(c) Do you think Steve Players is making money from this business?

(d) Would it be a wise investment of your money to buy this "system"?

Answer Key Added Practice 10 Red and Black Marbles and the Lottery

1. and 2. Neither Michael nor Erin will improve or hurt their chances of winning. Of the 12.3 million possible 6-number combinations, there are more combinations that have numbers spread out over the entire range than there are combinations that have consecutive numbers or combinations that have numbers all less than 32. So the probability that the winning combination will have spread-out numbers is higher than the probability that the winning combination will have all consecutive numbers, or the probability that the winning combination will have numbers all less than 32. However, that is not what players are betting on in a lottery. They are only allowed to bet that the one 6-number combination they chose will be the one that is randomly selected. Knowing that the winning combination is more likely to be "spread out" does not increase their chances of guessing the right combination. The following analogy may make this concept more understandable: Suppose there are 100 people in your neighborhood and one person was going to be chosen at random. Your neighborhood is 60% female and 40% male. If you are betting on what the selected person's gender will be, then you would bet female, since there is a 60% chance that the person will be female and a 40% chance that the person will be male. However, if you we are betting that a specific individual would be picked (for example, your mother), the person's gender would have no effect on their chances of being picked. One person is being selected randomly, so each person has an equal chance of being selected.

3. John's sister is wrong. The outcomes of lottery drawings are not dependent events. Each outcome is random, and previous outcomes do not have any effect on future outcomes. The random number generator from which the numbers are chosen does not know who John is related to, where he lives, or any other information about him. It is a machine, and it simply draws numbers. John's chances of winning

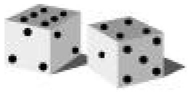
the lottery are the same before or after his sister, or any other person, wins the lottery.

4. (a) Some people who see this advertisement may believe that Steve Players is simply trying to trick customers through false statements and deceit, and that there is no truth to the claims made by the advertisement. Others may believe that the system actually works, and that they will win the lottery if they buy the system and use it. Most likely, both of these interpretations are incorrect. There is likely some degree of truth to the advertiser's claim. The book that you would receive if you sent in your money most likely states that you are guaranteed to win *some* lottery prize (for example, a $1 scratch ticket prize or matching 1 out of 4 on the daily numbers). When the advertiser guarantees that you will "win your state's lottery game," he is probably not referring to the lottery jackpot.

(b) If you buy the tickets for the full 18 days, you will spend $5 x 18 = $90. In addition, the book costs $35, so your total expenses would be $90 + $35 = $125.

(c) Steve Players is most likely making money from this business. Many of those who buy the system would probably win some small prize as a result of all of the lottery tickets they had to buy. However, the large majority of these "winners" will be losing money since the chances of winning a $1 prize or a $5 prize are far greater than the chances of winning a prize of $125 (to break even), or a prize of more than $125 (to make a profit). Most likely, anyone who won any prize, no matter how small, would be ineligible to get a refund. In addition, the majority of those who won nothing probably would not demand a refund due to embarrassment over being duped.

(d) This system would not be a wise investment of your money, since the large majority of people who use this system probably lose money.



Activity 11 Realities of Winning the Lottery

Objectives

- Understand applications of probability to social contexts
- Determine the effect of taxes and inflation rates on the income from lottery winnings
- Calculate annual interest
- Use logical reasoning to make informed decisions

Materials	paper, pencils, calculators
Time	45–60 minutes
Math Idea	States pour large sums of money into advertising the lottery—promoting quick money, instant wealth, and the end of poverty and anxiety about money and the future. Although many think about how picking the "lucky" combination will change their lives, few understand the financial and mathematical realities of winning the lottery.

Prior Understanding

Students should know how to work and calculate with percents and decimals, as well as use and evaluate expressions involving exponents.

Introduction: Gambling Connection

Pose the following question to students and discuss their opinions. After they do the activity, ask the question again.

Suppose you did win a $1 million lottery jackpot, are you really set for the rest of your life?

Discussion

To introduce the problem, have students complete the following sentence: "If I won the lottery, I would…." Keep track of some of the students' ideas on the board. Have students re-evaluate their answers after they have completed the activity. Some other questions students might consider include: If you won the lottery, would you still want to pursue a career? Could you be happy without a job for the next twenty years? What would happen after twenty years when the payments stopped?

Exercise 1

Explain that few states pay out lottery winnings in a lump sum. Most distribute the winnings in the form of an **annuity** that pays the winner a fixed amount a year over a span of 20 years (sometimes longer). Have students use mental math to determine the amount of money they would receive each year if they won $1,000,000.

Then have students assume that the annual payment would place them in the 35% tax bracket and that this tax rate would stay the same for the entire 20-year period. Have them calculate their net annual income—how much they would have left each year after taxes, and how much of the $1 million they would they receive in all.

Discussion

Before taxes, students would receive 1,000,000/20 = $50,000 a year. Point out that although they won $1 million, they cannot invest that much at once, nor can they accumulate interest on that amount; in the meantime, the state *is* earning interest on the remaining $950,000.

At the 35% tax rate, students would pay (0.35)(50,000) = $17,500 in taxes each year, leaving them with 50,000 – 17,500 = $32,500 net income. Over 20 years, they would net (20)(32,500) = $650,000. Since lottery winnings are taxed on both federal and state levels, this number would be considerably less.

Exercise 2

Have students assume a steady inflation rate of 4% each year. Explain that this means each subsequent year the buying power of $1.00 decreases by 4%: if you have $1.00 to spend this year, you'll have the equivalent of $0.96 to spend next year, and so on. Have students calculate what their after-tax lottery income would be worth each year for the 20-year period. Then have them calculate the total worth of that income after the full payment has been made and find the average yearly amount.

Discussion

With a 4% annual rate of **inflation**, after the first year, each dollar is worth the equivalent of $0.96. So after taxes, students would get $32,500 the first year. However, the next year, that $32,500 would only be worth (32,500)(0.96) = $31,200. This amount would be worth (31,200)(0.96) = $(32,500)(0.96)^2$ = $29,952 the third year; $(32,500)(0.96)^3$ = about $28,754 the fourth year, and so on. In 17 years, the actual value of the lottery income would be $(32,500)(0.96)^{16}$ = $16,237 or just about half the value of the income the first year. In the final year, they would receive the equivalent of $14,964. Over the 20-payment period (19 years) they would have collected the equivalent of \$453,373—much less than the \$650,000 expected after taxes. This averages out to 453,373/20 = \$22,669 a year over the 20 years.

Exercise 3

Have students assume that the average person spends \$9.00 each week on lottery tickets. Have them calculate the total amount of money spent for lottery tickets for one year and over the course of 30 years. Discuss the likelihood that they would break even in that amount of time.

Discussion

If they spend \$9 a week for lottery tickets, they would have spent 9×52 = \$468 in one year and 468×30 = \$14,040 over 30 years. Although it is possible that they might make some of this money back over the 30-year period, it is not likely that they will break even, and highly unlikely that they would win more (based on the results of Activity 6 Winning and Losing the Lottery).

Exercise 4

Have students suppose they invested the same yearly amount of money at 6% interest for 30 years. Ask them to find the total after 30 years and compare it to the amount spent on lottery tickets.

Discussion

Calculate the total yield from an initial investment of \$468 at 6% annual interest for a period of 30 years.

If students invest the initial \$468 at 6%, they would earn (468)(.06) = \$28.08 and would then have

468 + (468)(.06) = 468(1 + .06) = 468(1.06) = $496.08 at the end of the first year. To this they would add $468 (the annual amount they would have spent on lottery tickets) and invest the total $468 + $496.08 = $964.08 at 6%. At the end of the second year, they would have (468 + 496.08)(1.06) = $1021.92 to which they would add $468 and invest the total at 6%. If they continue the pattern of adding $468 and multiplying the total by 1.06 for 30 years, they would have about $39,219.

By this time students should realize that even if they won a $1 million lottery, after taxes and accounting for **inflation** they certainly would not have enough money to live a life of leisure.

Activity 11 Realities of Winning the Lottery Teacher Support

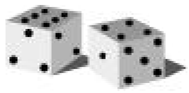
Vocabulary

annuity an investment that guarantees the owner a fixed payment each year for a specific number of years

inflation a decline in the value of money in relation to the goods and services it will buy

Ongoing Assessment

Suppose you had the option to have your $1 million prize as a lump sum or as payments of $50,000 over 20 years. Which would you choose? Why? *(Some students will realize that they could take the $1 million and invest it at 6% annual interest, giving them an income of $60,000 a year while still preserving the original $1 million. They would still have to pay taxes on the winnings the first year and on the interest income earned each year, but with wise investments at the end of 20 years, they would have more than the original million. Of course, they could also impulsively spend the money in the first few years and have nothing left. Some students may see the advantage of having smaller, but consistent payments over 20 years.)*



Added Practice 11 Realities of Winning the Lottery

Name ______________________________Date______________________

1. Susan will never forget the day her family and friends threw a big party celebrating the wonderfully unexpected event of her widowed mother winning the $35 million jackpot. Everyone seemed perfectly happy with the idea of her receiving a payment each year over the next 20 years, which was the arrangement set by the Lottery Commission. Two years later, Susan's mother was tragically killed in a car accident. As the next-of-kin, Susan receives a letter from the Internal Revenue Service letting her know that she owes 26% estate taxes on the rest of the money that was due her mother. How much money is this?

2. Five years ago you won $5 million, to be paid out over the next 20 years. This year you have spent a lot of money supporting your new passion for skiing and have had extremely high medical expenses. Stone Street Capital offers to buy your lottery annuity and give you a lump sum at the rate of 40¢ on the dollar. How much cash will you receive now and how does it compare with what you would receive if you continued to get some of your winnings every year? What are the advantages of choosing this option?

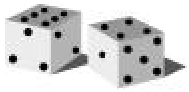
3. John won a $3 million lottery jackpot, which will be paid out over 20 years. The first year's after-tax payment totals $97,500. What percentage of his winnings is he paying in taxes?

Answer Key Added Practice 11 Realities of Winning the Lottery

1. Susan's mother would receive $35,000,000/20 = $1,750,000 a year. Assume that her mother collected 2 payments, or $3,500,000, before she died. There would be $35,000,000 – $3,500,000 = $31,500,000 remaining. At a tax rate of 26%, Susan would owe $31,500,000 × 0.26 = $8,190,000.

2. Your yearly payments are $5,000,000/20 = $250,000. You've already collected 5 payments or $1,250,000; 15 payments remain for a total of $3,750,000. If Stone Street Capital buys this remaining amount at 40 cents on the dollar, you will receive 0.40 × $3,750,000 = $1,500,000. It becomes a choice of having $1,500,000 now or $250,000 a year for the next 15 years. Selling the annuity to a company for one lump sum would provide a lottery winner with more money in the present but a smaller amount in the long run. This option would be advisable only if the lottery winner needed money immediately to pay off debts.

3. The pre-tax payment would be $3 million/20 = $150,000. Since John got $97,500, $150,000 – $97,500 = $52,500 was deducted in taxes; $52,500/$150,000 = .35 or 35%.



Activity 12 Work Behaviors of Lottery Winners

Objectives

- Develop and evaluate inferences and predictions that are based on data
- Use case studies to understand real-life situations involving probability
- Use logical reasoning to make informed decisions

Materials	paper, pencils, calculators
Time	30–45 minutes
Math Idea	H.R. Kaplan did a study on the effects of the lottery on the lives of lottery winners. Students will form a hypothesis about the work behaviors of lottery winners and use this study to test their hypotheses.

Prior Understanding

Students should know how to work and calculate with percents and decimals.

Introduction: Gambling Connection

Pose the following question to students and discuss their opinions. After they do the activity, ask the question again.

If a person does win a substantial amount of money in a lottery, how is his or her behavior affected? What factors influence the decisions lottery winners (and their spouses) make?

Exercise 1

Divide students into groups. Ask each group to form a hypothesis about the work behavior of lottery winners and their spouses. Students should consider what variables they think will influence winners' decisions. Have the group write its hypothesis on a sheet of paper along with reasons that they chose that hypothesis. For example, a group might hypothesize that lottery winners would quit their jobs because they would have enough money to last them the rest of their lives without working.

Distribute copies of BLM 12 to groups. Explain that the data show various work behaviors chosen by lottery winners based on a study done by H. R. Kaplan. Have students calculate the percent of winners and their spouses who chose each work option and fill in the results in the Percent (%) columns.

Discussion

Students should find that 56% of lottery winners do not change their work behavior after winning the lottery, while 11% quit their jobs; 62% of the spouses of lottery winners do not change their work behavior after winning the lottery, while 13% quit their jobs.

Exercise 2

Have students use the data from Exercise 1 to draw conclusions about what lottery winners and their spouses do after they win the lottery in relation to work behaviors.

Then have students determine whether the data seem to support the hypotheses they formed in Exercise 1. Discuss possible reasons why the data came out the way they did.

Discussion

Students should determine if the **hypothesis** they formed is supported by the **data** in the table. Encourage students to discuss the results of the study. For example, why wouldn't everyone quit their jobs if they won the lottery? What would be the advantages of maintaining the same work behaviors? Are the students surprised that 3% of the winners and 2% of the spouses of winners increased their hours of working after winning?

Students might **hypothesize** before seeing the **data** that age was an important variable affecting people's decision to remain in or leave the labor force. In fact, 39% of working winners 65 or older chose to retire early.

Other variables affecting work behavior are salary and educational level of winners. Workers earning less than $10,000 had the highest percentage of quitting and retiring. Winners with lower educational levels were more likely to quit their jobs, decrease their hours, and retire. These findings might mean that less-educated, lower-income individuals held jobs that were less satisfying and meaningful, and therefore, when given the opportunity, chose to leave their jobs more often than higher-educated, wealthier individuals. Another explanation could be that less-educated, lower-income individuals were willing to live only on the annual income the lottery provided because they were accustomed to living on a smaller income, whereas higher-educated, wealthier individuals could not support their present lifestyles on their annual lottery payments alone.

Activity 12 Work Behaviors of Lottery Winners

Teacher Support

Vocabulary

data information, often in the form of facts or figures obtained from experiments or surveys, used as a basis for making calculations or drawing conclusions

hypothesis a tentative explanation that accounts for a set of facts and can be tested by further investigation

representative sample a sample in which the characteristics of accurately reflect the characteristics of the whole population.

Ongoing Assessment

If response rate is defined as the percent of those who replied divided by the total number of people eligible to participate in the survey, what is the response rate of Kaplan's study of lottery winners? Do you think this a good response rate? Why or why not? *(A total of 576 usable questionnaires were returned. In addition, 20 potential respondents were found to be deceased, so the total number of eligible respondents was 2,319 – 20 = 2,299. Response rate = 576/2299 = 0.2505 or about 25.1%)*

Added Practice 12 Working Behaviors of Lottery Winners

Name ______________________________Date______________________

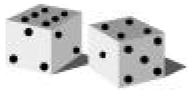
Answer each question based on what you know about the Kaplan study.

1. Do you think this is a **representative sample**? Explain.
2. Do you think there are any particular characteristics about the individuals who did not return the questionnaires? If so, what might these characteristics be?

Answer Key Added Practice 12 Work Behaviors of Lottery Winners

1. and 2. Student answers may include:

The sample of 2,319 could be representative of winners in the 12 states chosen. However, the information does not state how many states had lotteries in 1984, the year the study was conducted. If more than 12 states held a lottery in 1984, then the winners in the 12 sampled states might have different characteristics from the winners in the other states. In addition, only 25.1% of the sample returned their questionnaires, which is a small percentage in terms of being representative. Those who did not return the survey could differ significantly from those who did return the survey in age, feelings about working, income level, education level, and so on.



Blackline Master 12 Work Behaviors of Lottery Winners

Name ______________________________Date______________________

Questionnaires were sent to 2,319 lottery winners in 12 states between July and September of 1984. Winners included those who had won prizes ranging from $50,000 to millions. A total of 576 usable questionnaires were returned. In addition, the U.S. Postal Service returned 280 questionnaires because the addresses were not correct, and 20 potential respondents were found to be deceased.

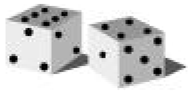
Type of Change in Work Behavior of Winners and Spouses After Winning

	Winners		Spouses	
Type of Change	**Number**	**Percent (%)**	**Number**	**Percent (%)**
Quit	49		34	
Retire	59		35	
Quit Second Job	10		3	
Work Fewer Hours	37		11	
Increase Hours	15		5	
Stayed Same	249		157	
Other	4		--	
TOTAL	446*	**	253*	**

* Not all respondents answered every question.

** May not total 100 due to rounding

[Data taken from Kaplan, H.R. (1987). Lottery winners: The myth and reality. *Journal of Gambling Behavior, 3,* 168-179.]



Answer Key Blackline Master 12

	Winners		Spouses	
Type of Change	**Number**	**Percent (%)**	**Number**	**Percent (%)**
Quit	49	11	34	13
Retire	59	13	35	14
Quit Second Job	10	2	3	1
Work Fewer Hours	37	8	11	4
Increase Hours	15	3	5	2
Stayed Same	249	56	157	62
Other	4	1	—	—
TOTAL	446*	**	253*	**

References

A hopeful sign. (1992, November). Life, 15, p. 21.

After 48 years, ex-wife sues for retroactive child support from ex-mate who won $4.1 million Lotto. (1994, August 8). Jet, pp. 32-34.

Arcuri, A.F., Lester, D., & Smith, R.O. (1985). Shaping adolescent behavior. Adolescence, 20, 935-938.

Bartecchi, C.E., MacKenzie, T.D., & Schrier, R.W. (1994). The human costs of tobacco use. New England Journal of Medicine, 330, 907-912.

Bartecchi, C.E., MacKenzie, T.D., & Schrier, R.W. (May, 1995). The global tobacco epidemic. Scientific American, p. 44-51.

Botvin, G.J. & Botvin, E.M. (1992). School-based and community-based prevention approaches. In J.H. Lowinson, P. Ruiz, R.B. Millman, & J.G. Langrod (Eds.), Substance abuse: A comprehensive textbook (pp. 910-927). Baltimore: Williams and Wilkins.

Carr, J.J. (1992). The art of science: A practical guide to experiments, observations, and handling data. San Diego: High Text Publications.

Chavira, R. (1991, February 25). The rise of teenage gambling. Time, p. 78.

Christiansen, E.M. (1993). The 1992 gross annual wager of the U.S.: industry abounds with 8.4% handle gain. Gaming and Wagering Business, 14, 12-35.

Clotfelter, C.T., & Cook, P.J. (1989). Selling hope: State lotteries in America. Cambridge: Harvard University Press.

Commission on the Review of the National Policy Towards Gambling. (1976). Gambling in America. Washington, D.C.: U.S. Government Printing Office.

Crites, T. (1994). Using lotteries to improve students' number sense and understanding of probability. School Science and Mathematics, 94, 203-207.

Dusenbury, L., Khuri, E., & Millman, R.B. (1992). Adolescent substance abuse: A sociodevelopmental perspective. In J.H. Lowinson, P. Ruiz, R.B. Millman, & J.G. Langrod (Eds.), Substance abuse: A comprehensive textbook (pp. 832-842). Baltimore, MD: Williams & Wilkins.

Eadington, W.R. (1992, October). Emerging public policy challenges from the proliferation of gaming in America. Paper presented at the Second Annual Australian Conference on Casinos and Gaming, Australia.

Eadington, W.R. (1996). Economic and social observations on youth gambling. Manuscript in preparation.

Ellickson, P.L. & Bell, R.M. (1990). Prospects for preventing drug use among young adolescents. Santa Monica, CA: Rand Corporation.

Ennet, S.T., Tobler, N.S., Ringwalt, C.L., & Flewelling, R.L. (1994). How effective is Drug Abuse Resistance Education? A meta-analysis of Project DARE outcome evaluations. American Journal of Public Health, 84, 1394-1401.

Facts on file. (1993). (Volume 53, No. 2749). New York: Facts on File. p. 584.

Gaboury, A., & Ladouceur, R. (1993). Evaluation of a prevention program for pathological gambling among adolescents. Journal of Primary Prevention, 14, 21-28.

Goodman, R. (1994). Legalized Gambling as a Strategy for Economic Development. United States Gambling Study.

Gould, L. (1995, April 23). Ticket to trouble. The New York Times Magazine, pp. 38-41, 54, 89-91.

Holman, R. (1995, July 10). My second question is the toughest. Lotto World, 3, pp. 10-11.

Houston bus driver cruising on 'easy street' after she won $41.48 million Texas Lotto jackpot. (1994, March 21). Jet, p. 7.

Huff, D. (1954). How to lie with statistics. New York: W.W. Norton & Company, Inc.

Kaplan, H.R. (1987). Lottery winners: the myth and reality. Journal of Gambling Behavior, 3, 168-179.

Kindt, J.W. (1994). The economic impacts of legalized gambling activities. Drake Law Review, 43, 51-95.

Ladouceur, R., Dubé, D., & Bujold, A. (1994). Gambling among primary school students. Journal of Gambling Studies, 10, 363-370.

Ladouceur, R., Gaboury, A., Dumont, M., & Rochette, P. (1989). Gambling: Relationship between the frequency of wins and irrational thinking. The Journal of Psychology, 122, 409-414.

Leonard, A. (1990, April 25). Teenage gambling. U.S. News and World Report.

Lesieur, H.R., & Klein, R. (1987). Pathological gambling among high school students. Addictive Behaviors, 12, 129-135.

Linn, E. (1993). Hitter: The life and turmoils of Ted Williams. New York: Harcourt Brace Javanovich.

Lotto World. (1995, July 10). Naples, FL: Lotto World, Inc.

Massachusetts Department of Education. (1995). Massachusetts curriculum frameworks for science and technology (Draft). Malden, MA: Author.

Massachusetts State Lottery Commission. (1994). A summary description of the Massachusetts state lottery. Braintree, MA: Author.

Mathematical Sciences Education Board. (1990). Reshaping school mathematics: A philosophy and framework for curriculum. Washington, D.C.: National Academy Press.

McGervey, J.D. (1986). Probabilities in everyday life. Chicago: Nelson-Hall, Inc.

McQueen, P. (1997). North American gaming report. International Gaming & Wagering Business (Suppl.). New York: International Gaming & Wagering Business.

Miller, E. (1995, July 10). How to enter the lottery zone. Lotto World, pp. 33-35.

Miller, J.D. (1989). Scientific literacy. Paper presented at the American Association for the Advancement of Science annual meeting, San Francisco, CA.

Morgan, K.O., Morgan, S., & Quitno, N. (Eds.). (1995). State Rankings 1995. Lawrence, KS: Morgan Quitno Corporation.

National Council of Supervisors of Mathematics. (1989). Essential mathematics for the twenty-first century: The position of the national council of supervisors of mathematics. Mathematics Teacher, 82, 338-391.

National Council of Teachers of Mathematics. (2000). Principles and Standards of School Mathematics. Reston, VA: Author.

National Council on Problem Gambling, Inc. (1993). The Need for a National Policy on Problem and Pathological Gambling in America. New York: The National Council on Problem Gambling, Inc.

National Research Council. (1989). Everybody counts: A report to the nation on the future of mathematics education. Washington, D.C.: National Academy Press.

National Research Council. (1994). National science education standards: Draft for comment and review. Washington, D.C.: National Academy Press.

National Science Foundation. (1992). The public understanding of science and technology in the United States, 1990. Report to the National Science Foundation. Washington, D.C.: Author.

Paulos, J.A. (1988). Innumeracy: Mathematical illiteracy and its consequences. New York: Hill & Wang.

Paulos, J.A. (1995). A mathematician reads the newspaper. New York: Basic Books.

Pellow, R.A. & Jengeleski, J.L. (1991). A survey of current research studies on drug education programs in America. Journal of Drug Education, 21, 203-210.

Rich, but not riche. (1994, May 30). People Weekly, p. 86.

Schmidt, W.H., McKnight, C.C., & Raizen, S.A. (1997). A Splintered Vision: An Investigation of U.S. Science and Mathematics Education. Michigan State University.

Shaffer, H.J. (1989, August 14). It's poor state policy to legalize gambling just for the revenue. The Boston Herald, p. 31.

Shaffer, H.J. (1994). The emergence of youthful addiction: The prevalence of underage lottery use and the impact of gambling. Massachusetts Council on Compulsive Gambling Technical Report #011394-100.

Shaffer, H.J., George, E.M., & Cummings, T. (1995). North American think tank on youth gambling issues: A blueprint for responsible public policy in the management of compulsive gambling. Boston, MA: Harvard Medical School.

Shaffer, H.J. & Hall, M.N. (1996). Estimating the prevalence of adolescent gambling disorders: A quantitative synthesis and guide toward standard gambling nomenclature. Journal of Gambling Studies, 12, 193-214.

Shaffer, H.J., Hall, M.N., & Vander Bilt, J. (1999). Estimating the prevalence of disordered gambling behavior in the United States and Canada: A research synthesis. *American Journal of Public Health, 89*, 1369-1376.

Shaffer, H.J., Hall, M.N., Walsh, J.S., & Vander Bilt, J. (1995). The psychosocial consequences of gambling. In R. Tannenwald (Ed.), Casino development: How would casinos affect New England's economy? Special Report No. 2. Boston: Federal Reserve Bank of Boston.

Shaffer, H.J., Stein, S.A., Gambino, B., & Cummings, T.N. (Eds.). (1989). Compulsive Gambling. Lexington, MA: Lexington Books.

Shaffer, H.J., Walsh, J.S., Howard, C.M., Hall, M.N., Wellington, C.A., & Vander Bilt, J. (1995a). Science and substance abuse education: A needs assessment for curriculum design (SEDAP Technical Report #082595-300). Boston, MA: Harvard Medical School Division on Addictions.

Shaffer, H.J., Walsh, J.S., Howard, C., Hall, M.N., Wellington, C., & Vander Bilt, J. (1995b). [Harvard/Billerica Addiction Science Education Project]. Unpublished raw data. Boston, MA: Harvard Medical School Division on Addictions.

Soukhanov, A.H. (Ed.). (1992). American Heritage Dictionary of the English Language (3rd ed.). Houghton Mifflin Company.

Svendsen, R., & Griffin, T., (1994.) Improving your odds: A curriculum about winning, losing, and staying out of trouble with gambling. Anoka, MN: Minnesota Institute of Public Health.

Veblen, T. (1899). The theory of the leisure class. New York: Macmillan Company.

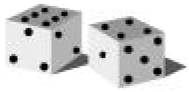
Vos Savant, M. (1995, June 11). Ask Marilyn. Parade, p. 15.

Wallack, L., Grube, J.W., Madden, P.A., & Breed, W. (1990). Portrayals of alcohol on prime-time television. Journal of Studies on Alcohol, 51, 428-437.

Walters, L.S. (1990, April 25). Education efforts aim at gamblers. Christian Science Monitor, p. 12, column 3.

Wheel of fortune. (1992, April). Life, 15, p. 20.

Zuckoff, M. (1995, September 27). Governor, Wampanoags reach accord on casino. The Boston Globe, pp. 1, 14.



Correlations

The correlations section presents mathematical concepts and the areas in the curriculum where those concepts are discussed. The correlations are divided into: Numbers and Number Relations, Algebra, Measurement, Geometry, Patterns, Relations and Functions.

Number and Number Relations		
N-1-M	demonstrating that a rational number can be expressed in many forms, and selecting an appropriate form for a given situation (e.g., fractions, decimals, and percents);	Thinking About Averages Number Sense Heads or Tails? Shared Birthdays Becoming a Legend You Bet Your Life! The Gambler's Fallacy Winning and Losing the Lottery Red and Black Marbles Realities of Winning the Lottery Work Behaviors of Lottery Winners
N-2-M	demonstrating number sense and estimation skills to describe, order, and compare rational numbers (e.g., magnitude, integers, fractions, decimals, and percents);	Thinking About Averages Number Sense Heads or Tails? Shared Birthdays You Bet Your Life! The Gambler's Fallacy Winning and Losing the Lottery Red and Black Marbles Realities of Winning the Lottery Work Behaviors of Lottery Winners
N-3-M	reading, writing, representing, and using rational numbers in a variety of forms (e.g., integers, mixed numbers, and improper fractions);	Thinking About Averages Number Sense Heads or Tails? Shared Birthdays Becoming a Legend

		You Bet Your Life! The Gambler's Fallacy Winning and Losing the Lottery Red and Black Marbles Realities of Winning the Lottery Work Behaviors of Lottery Winners
N-4-M	demonstrating a conceptual understanding of the meaning of the basic arithmetic operations (add, subtract, multiply and divide) and their relationships to each other;	Thinking About Averages Number Sense Heads or Tails? Shared Birthdays Becoming a Legend You Bet Your Life! The Gambler's Fallacy Winning and Losing the Lottery Red and Black Marbles Realities of Winning the Lottery Work Behaviors of Lottery Winners
N-5-M	applying an understanding of rational numbers and arithmetic operations to real-life situations;	Thinking About Averages Number Sense Statistics in Everyday Life Heads or Tails? Shared Birthdays Becoming a Legend You Bet Your Life! The Gambler's Fallacy Winning and Losing the Lottery Red and Black Marbles Realities of Winning the Lottery Work Behaviors of Lottery Winners
N-6-M	constructing, using, and explaining procedures to compute and estimate with rational numbers employing mental math strategies;	Number Sense Realities of Winning the Lottery

N-7-M selecting and using appropriate computational methods and tools for given situations involving rational numbers (e.g., estimation, or exact computation using mental arithmetic, calculator, computer, or paper and pencil);	Thinking About Averages Number Sense Heads or Tails? Shared Birthdays Becoming a Legend You Bet Your Life! The Gambler's Fallacy Winning and Losing the Lottery Red and Black Marbles Realities of Winning the Lottery Work Behaviors of Lottery Winners
N-8-M demonstrating a conceptual understanding and applications of proportional reasoning (e.g., determining equivalent ratios, finding a missing term of a given proportion).	Heads or Tails? Shared Birthdays Becoming a Legend You Bet Your Life! The Gambler's Fallacy Winning and Losing the Lottery Red and Black Marbles Realities of Winning the Lottery Work Behaviors of Lottery Winners
Algebra	
A-1-M demonstrating a conceptual understanding of variables, expressions, equations, and inequalities (e.g., symbolically represent real-world problems as linear terms, equations, or inequalities);	Heads or Tails? Shared Birthdays Becoming a Legend The Gambler's Fallacy Winning and Losing the Lottery Red and Black Marbles Realities of Winning the Lottery
A-2-M modeling and developing methods for solving equations and inequalities (e.g., using charts, graphs, manipulatives, and/or standard algebraic procedures);	Heads or Tails? Shared Birthdays Becoming a Legend Winning and Losing the Lottery

		Red and Black Marbles Realities of Winning the Lottery
A-3-M	representing situations and number patterns with tables, graphs, and verbal and written statements, while exploring the relationships among these representations (e.g., multiple representations for the same situation);	Number Sense
A-4-M	analyzing tables and graphs to identify relationships exhibited by the data and making generalizations based upon these relationships;	Thinking About Averages Number Sense Heads or Tails? Becoming a Legend You Bet Your Life! Work Behaviors of Lottery Winners
A-5-M	demonstrating the connection of algebra to the other strands and to real-life situations.	Thinking About Averages Number Sense Heads or Tails? Shared Birthdays Becoming a Legend The Gambler's Fallacy Winning and Losing the Lottery Red and Black Marbles Realities of Winning the Lottery Work Behaviors of Lottery Winners
Measurement		
M-1-M	applying the concepts of length, area, surface area, volume, capacity, weight, mass, money, time, temperature, and rate to real-world experiences;	Thinking About Averages Number Sense Winning and Losing the Lottery
M-2-M	demonstrating an intuitive sense of measurement (e.g., estimating and determining reasonableness of measures);	Thinking About Averages Winning and Losing the Lottery
M-3-M	selecting appropriate units and tools for	Thinking About Averages

	tasks by considering the purpose for the measurement and the precision required for the task (e.g., length of a room in feet rather than inches);	
M-4-M	using intuition and estimation skills to describe, order, and compare formal and informal measures (e.g., ordering cup, pint, quart, gallon; comparing a meter to a yard);	Thinking About Averages Winning and Losing the Lottery
M-5-M	converting from one unit of measurement to another within the same system (Comparisons between systems, customary and metric, should be based on intuitive reference points, not formal computation.);	Thinking About Averages Number Sense Winning and Losing the Lottery
M-6-M	demonstrating the connection of measurement to the other strands and to real-life situations.	Thinking About Averages Number Sense Winning and Losing the Lottery
Geometry		
G-1-M	using estimation skills to describe, order, and compare geometric measures;	
G-2-M	identifying, describing, comparing, constructing, and classifying geometric figures and concepts;	
G-3-M	making predictions regarding transformations of geometric figures (e.g., make predictions regarding translations, reflections, and rotations of common figures);	
G-4-M	constructing two- and three-dimensional models;	
G-5-M	making and testing conjectures about geometric shapes and their properties;	
G-6-M	demonstrating an understanding of the coordinate system (e.g., locate points, identify coordinates, and graph points in a coordi-	

	nate plane to represent real-world situations);	
G-7-M	demonstrating the connection of geometry to the other strands and to real-life situations (e.g., applications of the Pythagorean Theorem).	Winning and Losing the Lottery
Data analysis, Probability, and Discrete Math		
D-1-M	systematically collecting, organizing, describing, and displaying data in charts, tables, plots, graphs, and/or spreadsheets;	Thinking About Averages Number Sense Heads or Tails? Shared Birthdays You Bet Your Life! The Gambler's Fallacy Winning and Losing the Lottery
D-2-M	analyzing, interpreting, evaluating, drawing inferences, and making estimations, predictions, decisions, and convincing arguments based on organized data (e.g., analyze data using concepts of mean, median, mode, range, random samples, sample size, bias, and data extremes);	Thinking About Averages Number Sense Statistics in Everyday Life Heads or Tails? Shared Birthdays Becoming a Legend You Bet Your Life! The Gambler's Fallacy Winning and Losing the Lottery Work Behaviors of Lottery Winners
D-3-M	describing informal thinking procedures (e.g., solving elementary logic problems using Venn diagrams, tables, charts, and/or elementary logic operatives to solve logic problems in real-life situations; reach valid conclusions in elementary logic problems involving and, or, not, if/then);	Heads or Tails? Realities of Winning the Lottery Work Behaviors of Lottery Winners

D-4-M	analyzing various counting and enumeration procedures with and without replacement (e.g., find the total number of possible outcomes or possible choices in a given situation);	Heads or Tails? The Gambler's Fallacy Winning and Losing the Lottery Red and Balck Marbles
D-5-M	comparing experimental probability results with theoretical probability (e.g., representing probabilities of concrete situations as common fractions, investigating single-event and multiple-event probability, using sample spaces, geometric figures, tables, and/or graphs);	Heads or Tails? Shared Birthdays Becoming a Legend You Bet Your Life! The Gambler's Fallacy Winning and Losing the Lottery Red and Black Marbles
D-6-M	demonstrating the connection of data analysis, probability, and discrete math to other strands and to real-life situations.	Thinking About Averages Number Sense Statistics in Everyday Life Heads or Tails? Shared Birthdays Becoming a Legend You Bet Your Life! The Gambler's Fallacy Winning and Losing the Lottery Red and Black Marbles Realities of Winning the Lottery Work Behaviors of Lottery Winners
Patterns, Relations, and Functions		
P-1-M	describing, extending, analyzing, and creating a wide variety of numerical, geometrical, and statistical patterns (e.g., skip counting of rational numbers and simple exponential number patterns);	Number Sense Shared Birthdays The Gambler's Fallacy Realities of Winning the Lottery
P-2-M	describing and representing relationships us-	Number Sense

	ing tables, rules, simple equations, and graphs;	
P-3-M	analyzing relationships to explain how a change in one quantity results in a change in another (e.g., change in the dimensions of a rectangular solid affects the volume);	
P-4-M	demonstrating the pervasive use of patterns, relations, and functions in other strands and in real-life situations.	Number Sense Shared Birthdays The Gambler's Fallacy Realities of Winning the Lottery